MASS LOADING IN BACK-TO-BACK REFERENCE ACCELEROMETERS

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ABSTRACT

Mass loading on Back-to-Back Reference accelerometers can be a significant source of uncertainty in comparison calibrations. Modeling the effect of the mass of the unit-under-test on case resonance of the Reference accelerometer allows correction of these "relative motion" errors and reduction of uncertainty. Theory is given of mass loading on Reference accelerometers in general, and data is presented for the particular case of an integral transducer in a beryllium high frequency shaker.

INTRODUCTION

This paper will discuss mass loading and other frequency-dependent characteristics of Back-to-Back Reference Accelerometers. If ignored, they can cause significant errors and uncertainties in comparison calibrations. Errors and uncertainties can be minimized with proper understanding and correction. An example of correction of these effects is the use of the published mass loading correction curves of ENDEVCO® comparison standards. These curves assist in correcting for Reference accelerometer frequency response, as well as the less-well-understood mass loading effect. In this paper, data and theory are given behind the curves, as well as a description of how the data was obtained. Formulae for the correction curves are given, as they are applied in ENDEVCO's Automated Accelerometer Calibration System.

BACKGROUND

Comparison calibrations require that the motion applied to a Unit Under Test (UUT) be measured faithfully by a Reference accelerometer. Using the Reference output to determine input acceleration, the calculation of the UUT sensitivity is simply the UUT output divided by that acceleration level. In the simplest calculation, the Reference is assumed to have a fixed value of sensitivity, and both transducers are assumed to be experiencing the same acceleration.

At least two effects invalidate these assumptions, and are the topics of this paper. The first is the frequency response of the Reference itself. Of course, the Reference sensitivity is not a single value but is a function of frequency, typically increasing as the frequency approaches the resonance of the sensor assembly inside the Reference. The second is that there is flexibility in the structure between the transducers, allowing what is called "relative motion."

To ignore this flexibility implies belief that the structure between the transducers is infinitely stiff: that it does not deflect when transmitting the force required to accelerate the UUT. If this were so, the motions of the two transducers on each end would be identical. In reality any force on any structure will cause a deflection. The structure between the UUT and the Reference sensor responds to the forces by deflecting, therefore the motion at the UUT will not be the same as measured by the Reference. One moves "relative" to the other. There will be a difference in both phase and amplitude of their motions.

Usually the difference in motion is negligible, but particularly at high frequencies and/or with large values of UUT mass, it can be significant. In such conditions the forcing frequency may be near an undamped resonance of the structure. Deflections and calibration errors can become large.*

In the example of the Model 2270 Primary Comparison Standard, the structure is the upper part of the Case of the Reference accelerometer. Similarly, in the Model 2270M7A, built into the beryllium armature of the Model 2901 High Frequency Shaker, the structure is the material which physically connects the two accelerometers. These examples are depicted in simplified form in Figure 1.

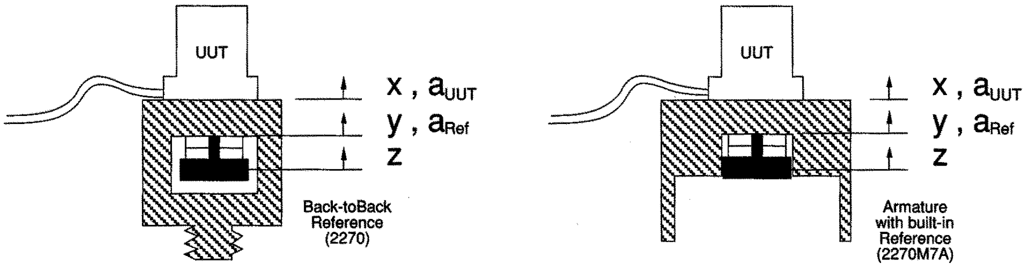

Figure 1. Two types of Back-to-Back Reference accelerometers. The "piggyback" style Endevco 2270 is depicted on the left, and the shaker armature with built-in 2270M7A is on the right.

Resonances in the structure between UUT and Reference will be referred to as the "Case resonance." For this discussion, the structure will be modeled as a one-dimensional spring between the accelerometers, with the Case resonance being the natural frequency of the spring. (A lumped parameter diagram is shown in Figure 2.) This resonance is determined by its stiffness and its distributed mass. Note that this is the resonance without the additional mass of the UUT.

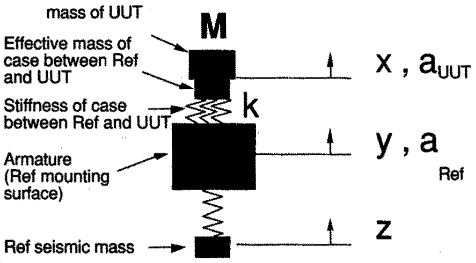

Figure 2. Lumped parameter model of Back-to-Back accelerometers. See text for definitions.

As shown in Figure 2, the motion x is at the UUT mounting surface, and is the motion which defines UUT sensitivity. The motion y is what is measured by the Reference. Z is the motion of the inertial mass of the Reference. The UUT is shown as a lumped mass as if its effect on the system were modeled with only the value of its mass, M. In fact the inertial mass and the stiffness elements in the UUT will cause the force required to drive the UUT to vary with frequency and complicate the mass loading discussion that follows. These effects will be ignored for frequencies below such resonances, since variations will occur only near UUT resonances, at which presumably the accuracy of mass loading corrections are less important.

*No discussion will be made of differences in direction of the motion, such as if rotations are involved in the relative motion. Only single-axis motion will be assumed, along the line between the transducers. In addition, the discussion will be limited to UUTs below their resonance frequencies, and mounted directly to a Back -to-Back Reference. Any fixtures mounted between the accelerometers or any large-amplitude resonances of the UUT in the frequency band of interest will invalidate the correction models described in this paper. The theory can be applied generally, however any correction values would need to take into account fixture mass and stiffness and the mechanical impedence of the UUT.

THEORY

Referring to Figure 2, the displacement of the base of the UUT is given by the value x, and the acceleration is given by its second derivative $\ddot{x}$. This is the motion which the UUT senses. Output is the product of that acceleration, the sensitivity, and any gain introduced by signal conditioning, according to

$$UUTOutput = \ddot{x}\ UUTSens\ (\omega)\ UUTGain\ (\omega)$$

In this expression the sensitivity and gain can vary with frequency ω.

To determine the UUT sensitivity, once the output is measured accurately and the gain (and frequency response) of the signal conditioning is known, only the acceleration remains to be found. In comparison calibration, of course, that is the purpose of the Reference transducer. Referring again to Figure 2, its output is related to input by

$$RefOutput = \ddot{y}\ RefSens\ (\omega)\ RefGain\ (\omega)$$

Note that its sensitivity too can vary with frequency. We will model the response as an undamped single degree of freedom system, given by

$$RefSens(\omega) = \frac{RefSens_o}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)}$$

At frequencies low compared to the natural frequency of the Reference, ω_n, the sensitivity is very close in value to $RefSens_o$. As frequency approaches the resonance, the value of the denominator becomes smaller and sensitivity increases in the classic amplification curve.[**]

One of the most important points of this paper is that the acceleration measured by the Reference is $\ddot{y}$, not $\ddot{x}$. Therefore, what is needed is a relationship to determine from y the motion of x, described below.

As mentioned above, the Case will be considered to be a spring with distributed mass. The distributed mass can be modeled as a lumped mass at the end of the spring (which in a simple linear model has an effective mass equal to one third of the total)[1]. The mass of the UUT is added to the effective mass, and its motion relates to the stiffness k of the Case according to the relation

$$(M_{eff} + M)\ \ddot{x} = k\ (y - x)$$

Leaving the details to the engineering textbooks (using the Laplace transform, which effectively substitutes $\ddot{x}$ with $-\omega^2 x$ by assuming the solution to be the complex periodic $x = X e^{-j\omega t}$), this expression can be solved and rearranged to get the transmissibility of the Case

$$\frac{x}{y} = \frac{k}{k - (M_{eff} + M)\ (\omega)^2}$$

The ramifications of this expression will be easier to see after a few more substitutions are made. Consider that the unloaded Case resonance (that is, the resonance frequency of the Case with no UUT) is defined by

$$\frac{k}{M_{eff}} = \omega_o^2$$

determined by the effective mass and stiffness of the Case. A further substitution is made with a mass loading ratio

$$\mu = \frac{M + M_{eff}}{M_{eff}} \qquad \textbf{(Eq. 1)}$$

which represents the multiplier by which the Case is loaded with the mass of the UUT. Making these substitutions gives

$$y = x\left(1 - \mu\left(\frac{\omega}{\omega_o}\right)^2\right)$$

This is the desired relationship between motion at the UUT and the Reference. As with the expression for the behavior of the Reference frequency response, at very

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[**] The fact that a Reference is not ideally flat should not be a detriment to calibration accuracy. If it is stable (that is, if the resonance frequency does not change) this curvature is simply a fixed characteristic that can be part of the overall calculation of the UUT sensitivity. Generally the actual sensitivity variations of the Reference are measured by techniques traceable to absolute standards. For those frequencies above which there are no traceable sensitivity data, we can use the model above. Regarding traceability, unlike the described mass loading corrections, compensating resonance response of Reference transducers generally is useful in frequency ranges which are too high to be traceable. Is the reduction of uncertainty by such methods moot because the measurement is not strictly traceable? An important function of a calibration system is in the characterization of transducer performance, such as the resonance search. To include corrections for well-behaved characteristics, such as a resonance rise of the Reference transducer, simply makes good engineering sense.

low values of frequency, the two motions are very nearly the same, and at frequencies approaching the modified Case resonance (altered by the added mass) the motions differ. The motion at the Reference sensor may be very much smaller than that out on the end of the resonating spring, which is the Case with the UUT attached.

Finally, combining all the formulae gives Equation 2, in which UUT sensitivity is equal to the ratios of the outputs and the gains multiplied by the Reference accelerometer sensitivity and a correction factor. Provided the frequencies are low enough, each of the frequency dependent terms in the correction factor will have a value less than one. In the numerator (on top) is the correction due to the Case resonance and mass loading; in the denominator is the effect of the natural frequency of the Reference.[***]

(Eq. 2)

$$UUTSens = \frac{UUTOutput}{RefOutput}\;\frac{RefGain}{UUTGain}\;RefSens_o\;\frac{\left(1-\mu\left(\frac{\omega}{\omega_o}\right)^2\right)}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}$$

Having a value less than one, the upper term corrects the value of calculated sensitivity downward. This counteracts the effect of the relative motion, which is to inflate the UUT output by creating motion larger than is measured by the Reference. The larger motion is the result of the resonant amplification due to the Case resonance. Larger UUT mass increases the need for correction, since larger UUT mass (larger μ) lowers the Case resonance. At a particular frequency, the motion of the heavier UUT would be farther up the amplification curve, requiring smaller absolute value of the correction term.

The Reference correction term also has a value less than one, but this time effects an increase in sensitivity because it is in the denominator. Since the actual Reference output becomes larger due to its crystal resonance ω_n, (that is, motion at z is larger than at y) if uncorrected it would result in a calculated sensitivity that is too small. The term corrects for this inflated Reference output.

The contributions of these two competing terms are depicted in Figure 3. A qualitative summary of the effects is listed in Table I.

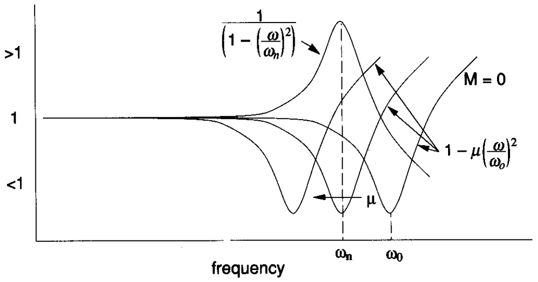

Figure 3. Frequency response contributions of the terms of the correction equation. The upper curve shows the effect of the Reference transducer resonance, peaking at ω_n. The three curves below show the effect of increasing UUT mass. The unloaded curve is on the right (M=0), and the curves with increased UUT mass to the left. The combined effect is the product of the top curve with the appropriate lower curve.

Note in the curves that there are conditions in which the competing effects of sensor resonances and mass loading might cancel, suggesting that Back-to-Back Reference accelerometers could be designed to have a Case resonance with a typical UUT which is matched to counteract the crystal resonance. However, as will be seen in the data below, the required Case mass would be substantial, reducing the drive capability of the shaker by a significant factor. Also, this would only work for a narrow range of UUT mass. It is thought to be better for the sake of overall uncertainty to keep the size of the correction to a minimum, considering the uncertainty of the correction itself is not insignificant.

DATA

Three sets of tests were performed, on one ENDEVCO 2901 High Frequency Standard-Shaker and two 2270M15 Accelerometers. The M15 is a back-to-back transducer with a 1.8 inch (46 mm) diameter solid beryllium case, structurally similar to the 2901 Armature, with a sensor assembly identical to the 2270M7A inside the shaker. Each test used a miniature 2250AM1 transducer adhesively mounted to a beryllium stud in the UUT position. Prior to the test, it was characterized with transient techniques to have a resonance frequency of approximately 80 kHz, with no detectable minor resonances.

[***] As mentioned in the previous footnote, this term would be replaced by the actual Reference frequency response measurements at those frequencies traceable to absolute standards.

Table I.
Effects of Resonance and Transmissibility on Correction Factor

Condition	**Reference Resonance Effect** Ref output will be:	**Trans-missibility Effect** UUT output will be:	**Apparent (Uncorrected) UUT Sens** UUTOut/RefOut	**Correction Factor** multiply by Apparent Sens
low freq, low mass	negligibly increased	negligibly increased	correct as is	≈1 (no correction)
low freq, high mass	negligibly increased	moderately increased	slightly high	less than 1
med freq, low mass	slightly increased	negligible	slightly low	greater than 1
med freq, high mass	slightly increased	slight amplification	slightly high	effects tend to cancel
high freq, low mass	significantly increased	negligibly increased	significantly decreased	can be much greater than 1
high freq, high mass	significantly increased	significantly increased	will have large uncertainties	use for trend only

To simulate mass loading effects, rings were mounted around the UUT using #8 screws in a triangular hole pattern, as depicted in Figure 4. All rings had an inner diameter of 0.45" (11.4 mm) to accommodate the miniature UUT. Outer diameters of the rings ranged from 1.25" to 2" (32 to 50 mm), with thicknesses from 0.25" to 0.375" (6 to 9.5 mm), and all were made of a tungsten material (except the lightest one, made of aluminum). The intent of the rings was to allow the mounting of different masses to the shaker without having to dismount the UUT. Since the 2250AM1 was mounted only once per set of tests, base strain, transmissibility of the adhesive, and other characteristics pertaining to mounting were eliminated as sources of uncertainty.

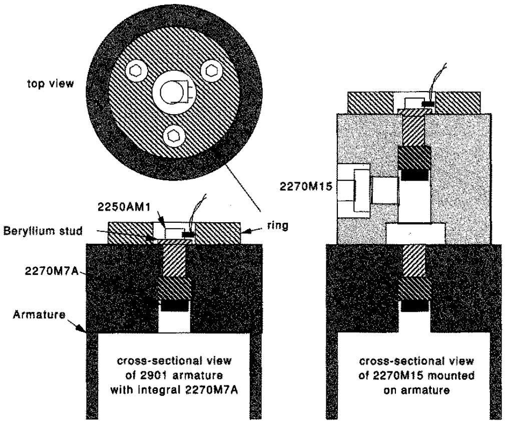

Figure 4. Configuration of the tests. The 2250AM1 was adhesively mounted to a beryllium stud and remained untouched throughout each set of tests. Rings of different mass were bolted to the armature or the 2270M15 to simulate different masses of UUT. When testing the 2270M15, the transducer in the shaker was not used.

Using the output of the 2250AM1 as the Reference, and correcting for its resonance rise using the classic single-degree-of-freedom equation, frequency response plots were generated for each attached mass. The data on one of the tests is given in Figure 5. As seen in the data, deviations of the curves below approximately 20 kHz is generally well below 1%. At 20 kHz and above, particularly if simulated UUT mass was greater than 100 grams, deviations appear to be on the order of 1/2 of the correction values. At least some of the variability, also apparent in the other two sets of data, may be due to minor connector resonances in the 2270 Series transducers.

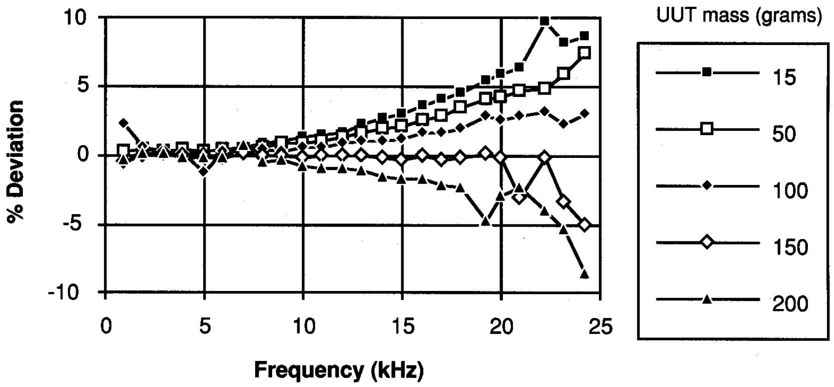

Figure 5. Sensitivity deviations of the 2270M7A Accelerometer. Data plotted in this figure are the sensitivity deviations of the Back-to-Back standard for different simulated masses, compared to an Endevco 2250AM1 transducer in the UUT position as the Reference. The adhesively-mounted miniature transducer was not removed between tests as masses were changed.

It should be noted that the data indicates strain sensitivity of the transducers was not a factor. The deviations at low frequency were essentially zero for all applied masses. A discussion of strain sensitivity is given in Reference [2].

Figure 6 below includes the curves generated to fit the data of Figure 5, by choosing values of Crystal resonance, Case resonance, and effective mass of the Case.

For comparison, a copy of the published correction curves from the 2270M15 Performance Specification is shown in Figure 7.

Justification or rationalization comes from the physically reasonable values used in the calculated curve, based on characteristics and dimensions of the structures. However, recent data have not been taken to confirm the published 2270 correction factors. (The data for the 2270 curves simulated with these coefficients predate the author.)

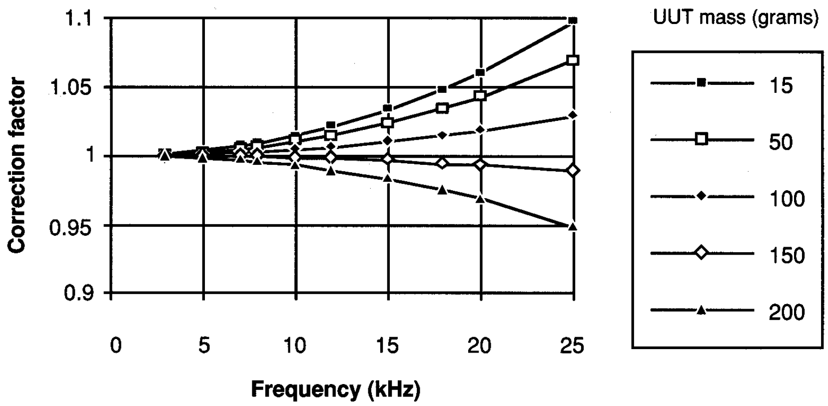

Figure 6. Calculated correction curves for the 2270M7A and 2270M15 accelerometers. Values of Effective Mass, Case Resonance, and Crystal resonance which provided the best fit using **Eq. 1** and **Eq. 2** are found in Table II.

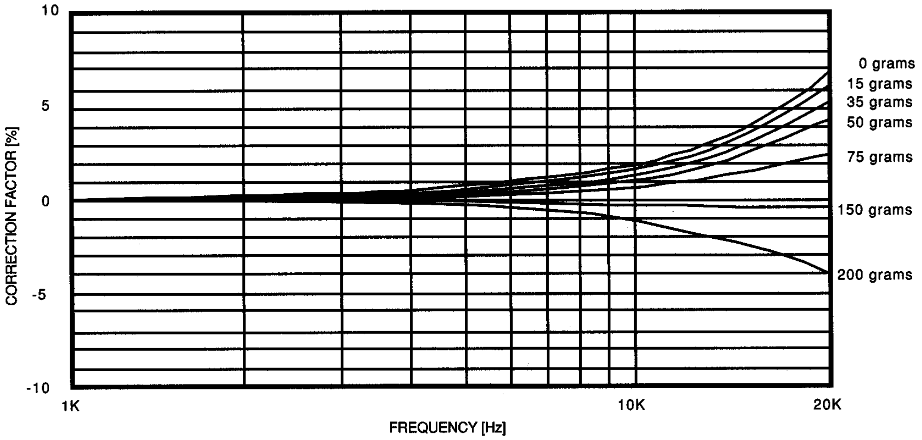

Figure 7. Published correction curves for the 2270M7A and 2270M15 accelerometers. These curves are found in the Performance Specifications for these Back-to-Back Reference accelerometers. The user of these accelerometers would multiply the frequency response results of a UUT by the appropriate curve to get a response corrected for the UUT mass.

Table II.
Parameters for calculation of correction factors.

Reference Transducer	**Effective Mass** (grams)	**Case Resonance** (kHz)	**Crystal Resonance** (kHz)
2270	6	130	50
2270M7A (or 2270M15)	6	380	78

Note that because the effective mass of the Case of the Reference is low, even lightweight UUTs can significantly change the loaded Case resonance. Only when the unloaded Case resonance is much higher than the UUT usable frequency range is this effect small. Note also the significant difference between the 130 kHz Case resonance of the steel case of the 2270 and the 380 kHz for the beryllium Case of the 2270M7A and 2270M15. This extraordinary value is reasonable, considering the fact that the speed of sound in beryllium (equal to the square root of the ratio of Young's modulus to density) is nearly 2.5 times as large as that of steel. (Beryllium is 40% stiffer but more than 4 x lighter than steel.) At nearly 500 000 in/sec (12 500 m/sec), a wave could travel between the Reference sensor and the UUT (across a distance of approximately 0.6 inch or 1.5 cm) approximately 500 000 / (2 x 0.6) or 417 000 times per second. This remarkable stiffness and velocity explains why such an exotic material is used.

CONCLUSIONS

Mass loading effects on Back-to-Back Reference accelerometers can follow predictable and physically reasonable patterns. They can be modeled, and therefore corrected, knowing the total mass of the UUT and the values of the Case resonance and the effective mass of the Reference. This technique can be used to reduce calibration uncertainty when using Back-to-Back Reference accelerometers. Whereas uncertainty estimates in the past were made unnecessarily large to swallow the relative motion effect of a range of UUT masses and frequencies, (or the estimates were subject to limits to UUT mass and frequency), use of the equations and modeling parameters described in this paper can reduce the size of the contribution of mass loading effects on overall uncertainty.

Future work is planned, using laser interferometric techniques to monitor motion at the UUT mounting surface with an absolute method, again using simulated UUT masses. The intent is to reduce the uncertainty of the correction, and provide more direct traceability.

ACKNOWLEDGMENTS

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BIOGRAPHY

Bob Sill received his Bachelor of Science and Master of Engineering degree in 1977 from Harvey Mudd College and joined Endevco in 1978. He is a Senior Project Engineer in the calibration systems group, and has specialized in silicon microsensors and shock calibration techniques.

ENDEVCO

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1.Thomson, W. T., "Theory of Vibration with Applications," Prentice-Hall, Inc., New Jersey, 1972, p20.

2."Instruction Manual No. 101 for Piezoelectric Accelerometers," ENDEVCO.