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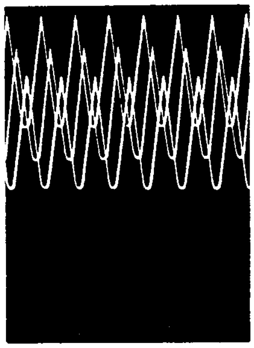

TP 219

BASIC SHOCK AND VIBRATION THEORY

by Dale Pennington

Shock and vibration are motions in mechanical systems. Vibration is an oscillating motion about a reference point. Shock is a transient motion. In many cases, these motions are undesirable, even destructive. In order to measure and control them, it is necessary to understand their basic natures.

Mechanical motions can be divided into two general classes: periodic motions, which repeat after a fixed time interval, and aperiodic, which do not. Shock, of course, is aperiodic. Vibration may be either.

Simple Harmonic Motion

The simplest form of periodic motion may be represented (Fig.1) as the projection of a rotating vector on a vertical axis as it moves with uniform circular motion (constant angular velocity ω).

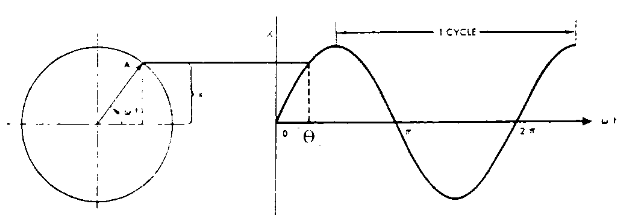

Figure 1

As **A** rotates, **x** varies in length. The variation is plotted against ωt. By inspection,

$$x = A \sin \omega t. \tag{1}$$

After completing one entire rotation (or cycle) of 2π radians, the wave repeats. The time required to accomplish one cycle is defined as the period (**T**) of the motion. Since for one cycle, $\omega t = 2\pi$, the period is clearly

$$T = \frac{2\pi}{\omega}. \tag{2}$$

Frequency (**f**) is the reciprocal of the period, or

$$f = \frac{\omega}{2\pi}. \tag{3}$$

Equation 1 is plotted as a function of time in Figure 2.

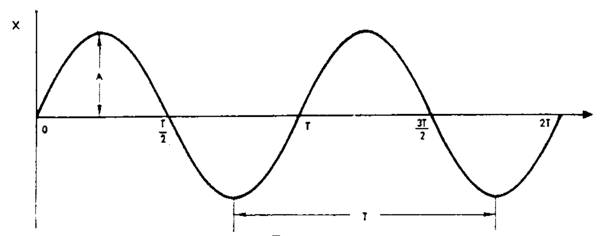

Figure 2

For the (more general) case where **x** is not zero when $t = 0$, the equation becomes

$$x = A \sin (\omega t + \phi), \tag{4}$$

where ϕ is known as the phase angle. Equation 4 describes a wave with the same frequency as $A \sin \omega t$ but which is displaced from it by ϕ degrees, or by ϕ/ω seconds.

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Using the method of rotating vectors, the relationship between two such functions is shown in Figure 3.

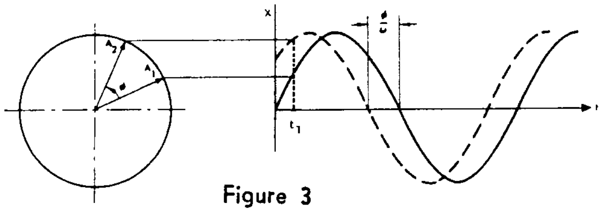


Figure 3

Equation 1 represents a vibratory displacement. The corresponding velocity and acceleration are determined by successive time differentiations.

$$v = \frac{dx}{dt} = \omega A \cos \omega t$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \sin \omega t$$

Using trigonometric identities, these can be rewritten as:

$$v = \omega A \sin (\omega t + \pi/2)$$

$$a = \omega^2 A \sin (\omega t + \pi)$$

These equations show that the velocity and acceleration are also sinusoidal functions. They are of the same frequency as the displacement, but displaced along the time axis. Velocity leads the displacement by a phase angle of $\pi/2$, or 90°. The acceleration leads by 180°. (These quantities may, of course, also be represented by rotating vectors, as in Figure 3.)

For any sinusoidal wave, the definitions of Figure 4 apply. Equivalent values are:

rms = .707 × peak
Average = .637 × peak
Peak to peak = 2 × peak

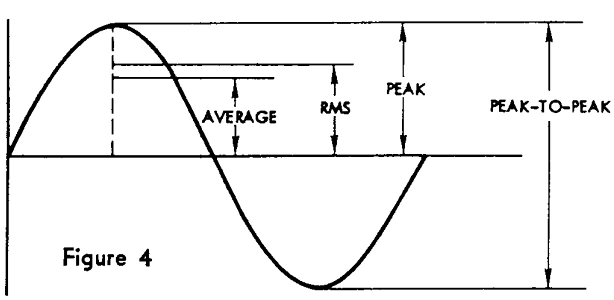


Figure 4

As an example, suppose the peak value of the function of Figure 4 is 10 g (where one "g" is the acceleration of gravity = 32 ft/sec^2 = 386 in./sec^2). It can also be expressed as:

10 g vector
10 g maximum
± 10 g
7.07 g rms
6.37 g average
20 g peak-to-peak

All are equal to 10 g pk.

Complex Motion

Although harmonic motion is periodic, not all periodic motion is harmonic. Figure 5 is an example of a complex wave whose motion is periodic.

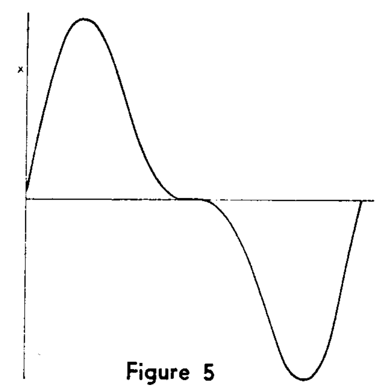


Figure 5

Analysis of complex waves would be very difficult but for a relationship proved by Fourier which shows that all periodic waves can be represented as a sum of sinusoidal, or simple harmonic waves. (The frequency of the component waves are harmonically related: *i.e.*, they are simple multiples of the fundamental frequency of the complex wave in question.) This representation is known as a Fourier Series; and is generally written:

$$x = A_1 \sin (\omega t + \phi_1) + A_2 \sin (2\omega t + \phi_2) + \ldots$$

or

$$x = A_0 + \sum_{k=1}^{n} A_k \sin (k\omega t + \phi_k). \quad (5)$$

For example, the complex wave of Figure 5 can be shown to be the sum of two harmonically related sinusoids:

$$x_1 = A_1 \sin \omega t \qquad x_2 = A_2 \sin 2\omega t$$

(in this instance, $\phi = 0$). Algebraic addition yields

$$x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin 2\omega t,$$

which is the equation of the curve in Figure 6. (The function x can be plotted by graphical addition of curves x_1 and x_2.)

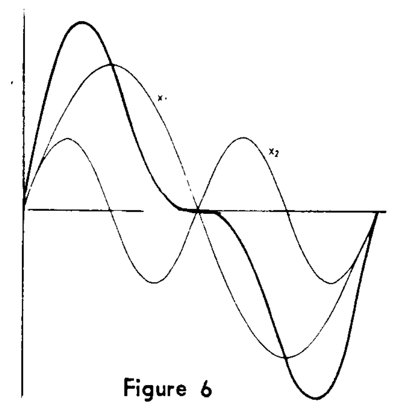


Figure 6

It should be pointed out that the simple relationships between displacement, velocity and acceleration found for simple harmonic motion do not apply in complex motion. The velocity and acceleration functions that can be determined by graphical differentiation of the displacement or by differentiating the Fourier series will be seen to possess an entirely different wave shape than the related displacement.

It is also important to note that the simple numerical relationship between peak, rms and average in harmonic motion values are not valid for complex waves. For complex waves, more rigorous definitions must be used:

$$x_{Average} = \frac{1}{T} \int_0^T x(t)\,dt \qquad (6)$$

$$x_{rms} = \sqrt{\frac{1}{T} \int_0^T [x(t)]^2\,dt}\;. \qquad (7)$$

Dynamics: Free Vibration of an Undamped Single Degree of Freedom System.

A simple spring-mass system is shown schematically in Figure 7. The mass **m** is free to move vertically and is attached to a fixed (immovable) support by spring **k**. The position of **m** is completely described by its distance along the vertical axis. (Since only one coordinate, **x**, is needed to describe the location **m**, the system is defined as a single degree of freedom system.)

If the mass is displaced a distance **x** from its equilibrium position* and released, the only force acting on it is the elastic restoring force of the spring, **−kx**, where **k** is the spring stiffness. The resultant motion is governed by Newton's second law, F = ma, which becomes −kx = ma, or:

$$-kx = m\frac{d^2x}{dt^2}\;.$$

* When the mass is first attached, the spring will stretch a small amount δ, known as the static deflection. At equilibrium, the effect of gravity on **m** is just balanced by the restoring force of the spring, or, $W = -k\delta$. (The minus sign is introduced since the force acts opposite to the deflection δ.)

Rearranging terms:

$$m\frac{d^2x}{dt^2} + kx = 0, \qquad \text{or}$$

$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0\;. \qquad (8)$$

A general solution of this differential equation is

$$x = A\sin\left(\sqrt{\frac{k}{m}}\,t + \phi\right),$$

which is seen to be the same form as Equation 4, the general equation for simple harmonic motion.

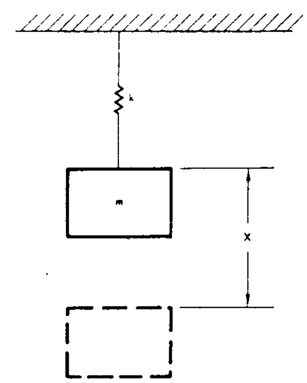


Figure 7

Therefore, the spring-mass system of Figure 7 undergoes sinusoidal vibration with an angular velocity of

$$\omega = \sqrt{\frac{k}{m}}\;. \qquad (9)$$

The period, from Equation 2, is:

$$T = 2\pi\sqrt{\frac{m}{k}}\,, \qquad (10)$$

and the natural frequency is:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}\;. \qquad (11)$$

The system described above will, once started, continue to vibrate at its natural frequency forever, unless some opposing force is introduced. For example:

> A 16-pound weight suspended from a helical spring produces a static deflection of 6 inches. What is the natural frequency of this system?

$$m = \frac{W}{g} = \frac{16 \text{ lb}}{32 \text{ ft/sec}^2} = .5 \text{ lb-sec}^2\text{/ft.}$$

$$k = \frac{W}{\delta} = \frac{16 \text{ lb}}{.5 \text{ ft}} = 32 \text{ lb/ft.}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{32}{.5}} = \frac{4}{\pi} = 1.27 \text{ Hz}$$

Free Vibration of a Damped Single Degree of Freedom System

Observation of actual vibrating systems, such as bells and tuning forks, shows that vibrations, once initiated, do not continue forever but eventually die out because of friction effects. We say that these vibrations are damped. If the mass of Figure 8 is displaced a distance **x** and released, its motion will be determined not only by spring **k**, but by the damper of damping coefficient **c**. The effect of the damper is to introduce a viscous damping force **−cV**, which is proportional to the velocity of **m** and which opposes that velocity. The equation of motion now becomes:

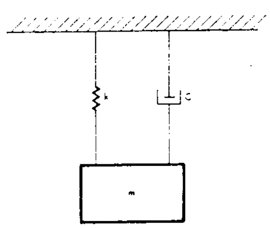


Figure 8

$$ma = -kx - cV$$

$$m\frac{d^2x}{dt^2} = -kx - c\frac{dx}{dt}$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

$$\frac{d^2x}{dt^2} + \left(\frac{c}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x = 0 \qquad (12)$$

The solution of Equation 12 depends on how much damping is present. Let us arbitrarily define the term critical damping c_c as $c_c = 2m\sqrt{k/m}$. Then any amount of damping can be discussed in terms of its ratio to critical damping. The damping ratio is commonly written as $\zeta = c/c_c$.

Less than Critical Damping

For the case of little damping ($\zeta < 1$) the solution of Equation 12 is:

$$x = Ae^{\frac{-ct}{2m}} \sin(\omega t + \phi), \qquad (13)$$

where $\omega = \sqrt{\frac{k}{m}(1-\zeta^2)}$

This is a sinusoidal vibration with diminishing amplitude as shown in Figure 9. (The rate of decay of the vibration amplitude is conveniently expressed in terms of the logarithmic decrement (Δ), defined as the natural logarithm of the ratio of any two successive peaks ($\Delta = \ln\frac{x_1}{x_2}$). In terms of ζ, this becomes $\Delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$).

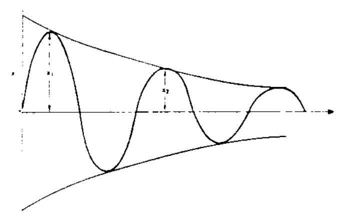


Figure 9

Critical Damping

When $c = c_c$ ($\zeta = 1$), the solution of Equation 12 becomes

$$x = (A + Bt)\,e^{\frac{-ct}{2m}}. \qquad (14)$$

In this case there is no oscillation and the motion is as shown in Figure 10. Critical damping can be described as that degree of damping for which the mass will return to its equilibrium position in the least time without oscillation.

Greater than Critical Damping

Where **c** is larger than c_c ($\zeta > 1$), the solution to Equation 12 is:

$$x = e^{\frac{-ct}{2m}}\left[Ae^{\omega t} + Be^{-\omega t}\right], \qquad (15)$$

where $\omega = \sqrt{\frac{k}{m}(\zeta^2 - 1)}$.

Again there is no oscillation. The mass returns to its equilibrium position more slowly than when critically damped (Fig. 10).

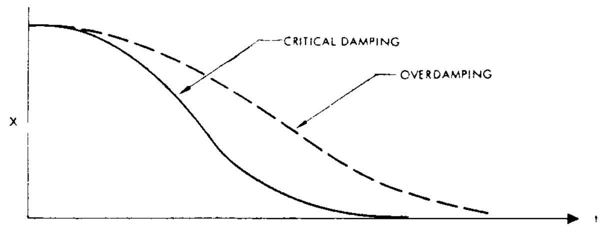


Figure 10

Forced Vibration of a Damped Single Degree of Freedom System

So far the only case considered is that for which the mass is displaced a given distance and released. The resulting free vibration eventually dies out because of energy dissipation in the damper. By supplying additional energy to the vibrating system, the amplitude of vibration can be maintained.

Consider the system of Figure 11, in which a sinusoidal forcing function **F** acts upon the mass. The equation of motion is:

$$ma = -kx - cV + F$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F$$

$$\frac{d^2x}{dt^2} + \left(\frac{c}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x = F_0 \sin \omega t \quad .(16)$$

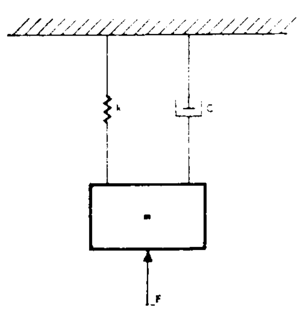


Figure 11

Allowing sufficient time for the system to settle down to steady-state oscillation, the solution of Equation 16 can be written:

$$x = A \sin(\omega t - \phi), \qquad (17)$$

where $A = \dfrac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$

and $\tan \phi = \dfrac{c\omega}{k - m\omega^2}$.

We introduce two new terms:

$\omega_n = \sqrt{\dfrac{k}{m}}$ = the undamped natural frequency

$A_{static} = \dfrac{F_0}{k}$ = the static deflection of the system due to a constant force F_0.

It is now possible to normalize the expressions for **A** and ϕ to the following:

$$R = \frac{A}{A_{static}} = \frac{1}{\sqrt{\left(1 - \left[\frac{\omega}{\omega_n}\right]^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} \qquad (18)$$

$$\tan \phi = \frac{2\zeta\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} . \qquad (19)$$

The relative response **R** (or magnification factor) and phase angle ϕ can now conveniently be plotted as functions of only two variables: the frequency ratio $\frac{\omega}{\omega_n}$ and the damping factor ζ (see Fig. 12 and 13).

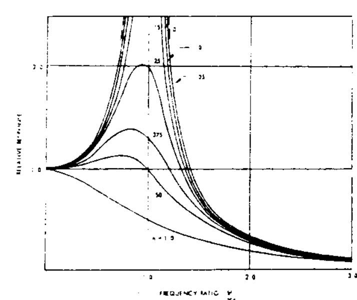


Figure 12

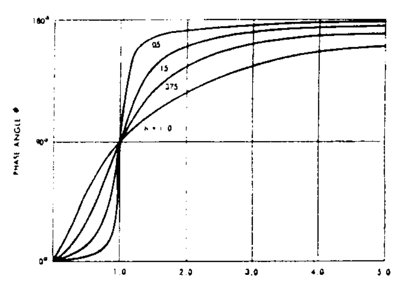


Figure 13

These curves show that both the vibration amplitude and the phase angle (between the forcing function and the resultant displacement of m) are strongly affected by the damping ratio and the frequency of vibration. In general, as damping becomes smaller amplitude of vibration at resonance becomes larger. Small values of damping are said to produce high "Q" resonance. (In theory, with zero damping the vibration amplitude could become infinitely large.) Also, for small damping the pase angle shifts more rapidly from 0° to 180°. For lightly damped systems, one technique for finding resonance is to determine the frequency of 90° phase shift.

Aperiodic Motion: Random Vibration

Random vibration is a continuous oscillating motion whose instantaneous amplitude can be predicted only on a probability basis. It may be considered as being composed of a continuous spectrum of frequencies whose individual amplitudes are varying in a random manner. Random motion is obviously aperiodic and is described mathematically in terms of statistics, rather than trigonometeric functions. Figure 14 is a plot of random acceleration vs. time. At any given instant (t), the probability that the acceleration value is between a_o and $a_o + da$ is defined as: p (a) da. For a normal process:

$$p(a) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{a^2}{2\sigma^2}\right) \qquad (20)$$

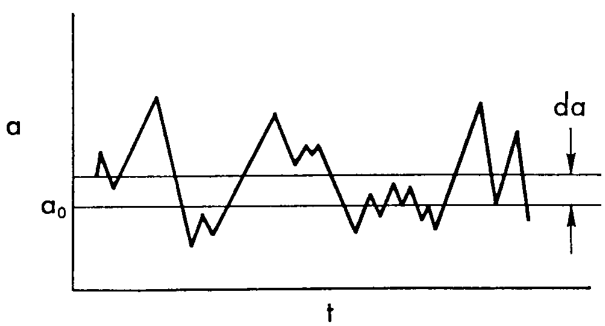


Figure 14

This equation, known as a Gaussian or Normal Distribution, is plotted in Figure 15 as a function of a.

The probability that the instantaneous value of acceleration is between a_1 and a_2 is given by:

$$\int_{a_1}^{a_2} p(a)\,da = \frac{1}{\sigma\sqrt{2\pi}} \int_{a_1}^{a_2} \exp\left(-\frac{a^2}{2\sigma^2}\right) da$$

which is the shaded area under the curve of Figure 15.

Probability Density

Since the amplitude probability is the product of p (a) and an acceleration value, p (a) is commonly referred to as the probability density. Equation 20, then, describes a Gaussian probability density function. (The only vibrations considered here will be those which are described by Equation 20. It will also be assumed that the random vibration is stationary -- that is to say that its statistical properties are unaffected by a translation of the time axis.)

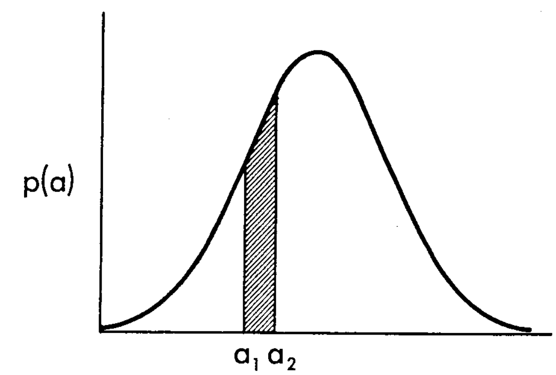


Figure 15

The quantity σ is defined as the root mean square deviation of the instantaneous acceleration from the mean acceleration value. For random acceleration, the mean acceleration value is zero, so that σ reduces simply to the rms value of the instantaneous acceleration.

The probability density curve is usually normalized -- that is, the scales are so adjusted that the total area under the curve is unity, which is to say that the total area under the curve represents certainty, with a probability of 1.

Referring to Figure 16, the probability that the instantaneous value of acceleration is between $\pm a_1$ is equal to the shaded area under the normalized probability density curve.

Random Amplitude Sine Wave

From a damage standpoint, peak values of acceleration may become more important than instantaneous values. Suppose the random vibration signal is passed through a narrow bandwidth filter. The result will be a single frequency wave with randomly varying amplitude, or random amplitude sine wave.

For such a wave, the probability of an acceleration peak having a value between a_o and $a_o + da_p$ is defined as:

$$p(a_p)\,da_p$$

Where

$$p(a_p) = \frac{a_p}{\sigma^2} \exp\left(-\frac{a_p^2}{2\sigma^2}\right) \qquad (21)$$

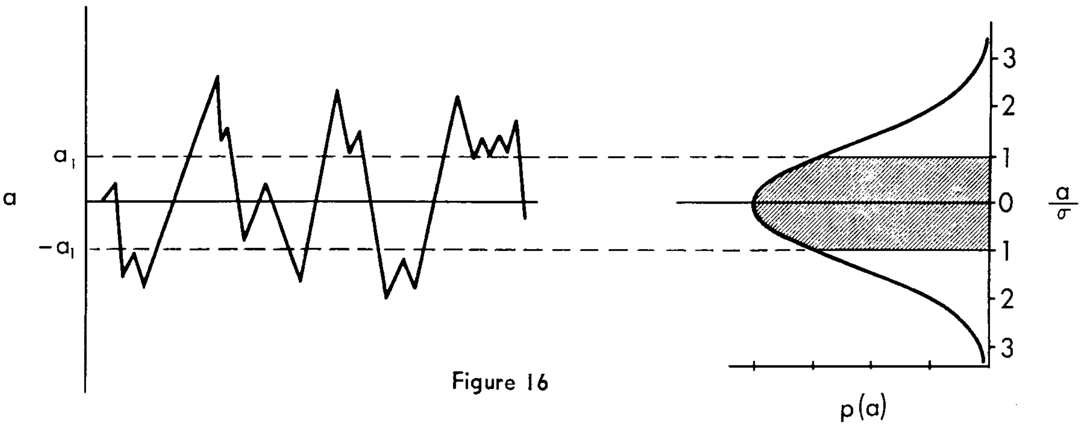


Figure 16

This equation, known as a Rayleigh distribution, is plotted in Figure 17 as a function of a_p.

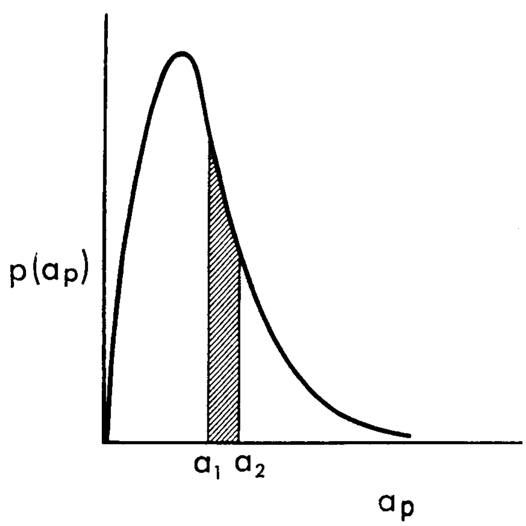


Figure 17

From Equation 21, the probability that the peak value of acceleration is between a_1 and a_2 becomes

$$\int_{a_1}^{a_2} p\,(a_p)\,da_p = \frac{1}{\sigma^2} \int_{a_1}^{a_2} a_p \exp\left(-\frac{a_p{}^2}{2\,\sigma^2}\right) da_p$$

which is the shaded area under the curve.

By analogy with the previous discussion of instantaneous acceleration, $p(a_p)$ is a probability density function for peak accelerations, or equivalently, of the envelope of the random sine wave.

Referring to Figure 18, the probability that the peak acceleration is between o and a_1, is equal to the shaded area under the normalized probability density curve.

A single frequency component of a random vibration will vary randomly in amplitude. This component cannot, therefore, be specified by its peak value. Instead, its root mean square (rms) value, which does not vary with time, must be used.

The rms value of a single frequency wave is easily defined. Random vibration, however, contains a continuous distribution of frequencies. Since any actual bandpass filter used in practice has a finite bandwidth, it will pass frequencies adjoining the center frequency of interest. To determine the contribution of a single frequency, it is necessary to divide the filter output by its bandwidth. If output is in rms gs, it is divided

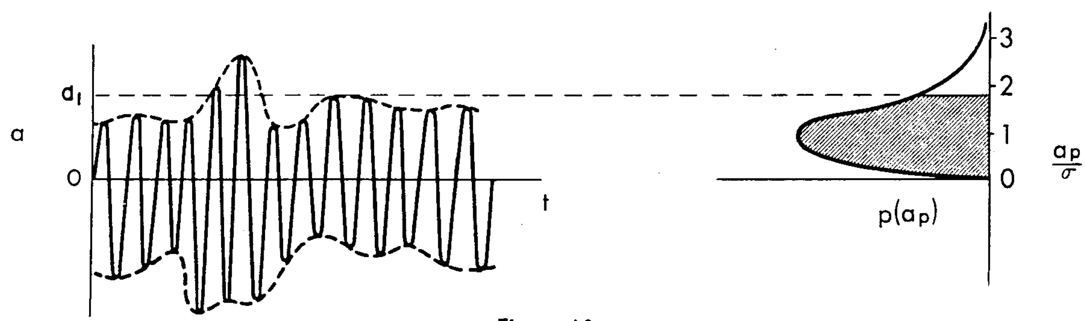


Figure 18

by the square root of the bandwidth (yielding $\frac{g}{\sqrt{Hz}}$). If output is in mean squared gs, it is divided by the bandwidth in Hz (yielding $\frac{g^2}{Hz}$).

Power Spectral Density Plot

Random vibration is normally plotted in the latter units as a function of frequency. Figure 19 illustrates a typical power spectral density plot. (Power, the rate of doing work, is proportional to the square of the vibration amplitude. Hence a plot $\frac{g^2}{Hz}$ vs. frequency shows the power distribution of the vibration as a function of frequency.)

The shaded area under the PSD curve is given by

$$\bar{a}^2 = \int_{f_1}^{f_2} G(f)\, df \qquad (22)$$

and represents the mean squared acceleration between f_1 and f_2.

The rms acceleration (between frequencies f_1 and f_2) is therefore equal to the square root of the shaded area of Figure 19.

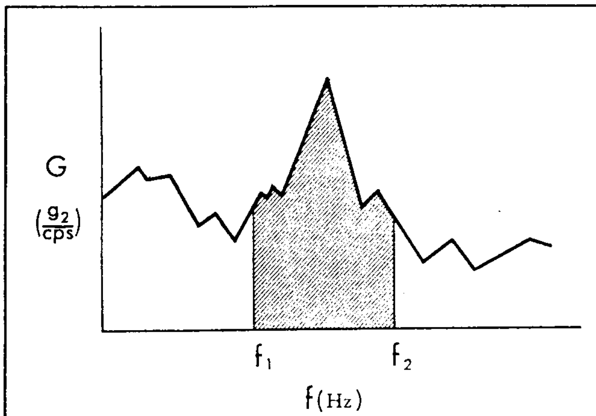


Figure 19

Random vibration which exhibits a constant acceleration density is called a white noise. In the case of white noise, Equation 22 simplifies to:

$$\bar{a}^2 = G_o B$$

and $$a_{rms} = \sqrt{G_o B} \qquad (23)$$

when G_o = constant acceleration density and B = bandwidth under consideration.

Random vibration is important because it it frequently encountered in nature. Rocket engines are typical random vibration generators. Fortunately, the rms acceleration level, which is easily measured, has statistical significance. The amplitude of random vibration is most often specified as an rms acceleration over a given bandwidth and as an acceleration density vs frequency.

Dynamics

The behavior of the system illustrated in Figure 8 (EQ, Sept., 1962, Pg 22) was examined for a forced vibration of the mass. If the forcing function is applied instead to the support, the resultant motion of the mass or transmissibility of the system becomes:

$$T = \sqrt{\frac{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}{\left(1 - \left[\frac{\omega}{\omega_n}\right]^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}} \qquad (24)$$

If the forcing function is random sine wave (single frequency) the resultant motion of the system is the same as for a pure sinusoidal input. The product of the input excitation and the system transmissibility is $a = a_o T$, where a = resultant system acceleration, a_o = input acceleration (of support), T = transmissibility.

In terms of squared acceleration:

$$\bar{a}^2 = \bar{a}_o^2\, T^2$$

where a^2 = mean squared resultant acceleration and $\bar{a}^2$ = mean squared input acceleration.

Since only a single frequency is present, $\bar{a}^2$ is equal to G(f), the mean square acceleration density. If, instead of a single frequency, the input is a random vibration the overall mean squared response is the sum of the mean squared responses to the component frequencies:

$$\bar{a}^2 = \int_0^{\infty} T^2\, G(f)\, df \qquad (25)$$

Simple Impulse

Mechanical shock, although difficult to define rigorously, can be expressed as a sudden, non-periodic disturbance involving relatively large motions in a system within a short (in relation to the natural period of the system) time interval. Since shock occurs in infinite variety and can be very complex, it is useful to examine several simplified types of shock motion as an introduction to more complex forms.

The simplest concept is that of a simple impulse, or velocity shock (Fig. 20). In this case the shock motion is an impulse of extremely short duration and large acceleration which results in a step velocity change. In the (ideal) limiting case the pulse duration approaches zero and the amplitude becomes infinitely large.

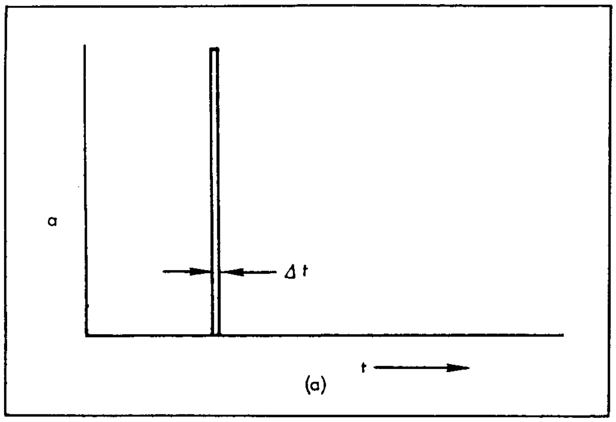
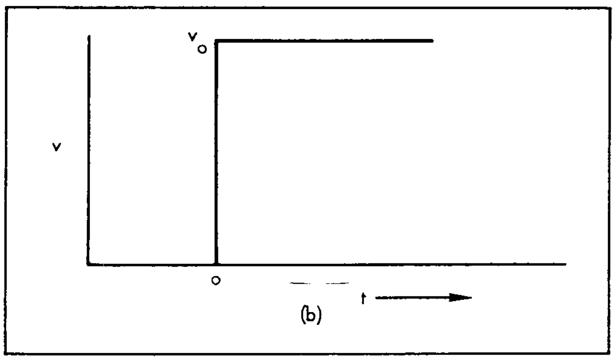


Figure 20. Impulse or velocity shock: (a) as a function of acceleration, (b) as a velocity-time function.



This (acceleration) motion is expressed analytically as:

$$a(t) = \lim_{\Delta t \to 0} \frac{v_0}{\Delta t} \qquad (26)$$

An alternative expression defines a(t) in terms of the Dirac function, $\delta(t)$ [defined as zero for $t \neq 0$ and infinite at $t = 0$ such that $\int_{-\infty}^{+\infty} \delta(t)dt = 1$] and is:

$$a(t) = v_0\, \delta(t) \qquad (27)$$

This type of shock is approached in collisions between very hard materials, such as steel impacting on steel.

Perhaps the most common type of shock encountered in test work is the simple pulse, specified by its acceleration amplitude, time duration and pulse shape. Examples are the half sine, square wave and sawtooth pulses.

The half sine pulse (Fig. 21a) has been used extensively. It is not only easy to generate but yields readily to mathematical analysis. It is expressed analytically as:

$$\begin{aligned} a(t) &= A \sin \frac{\pi t}{\tau} & [0 < t < \tau] \\ a(t) &= 0 & [t > \tau,\ t < 0] \end{aligned} \qquad (28)$$

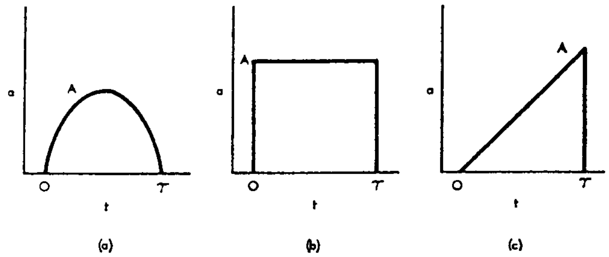


Figure 21. Simple pulse shapes: (a) half sine, (b) square wave, (c) sawtooth (terminal peak).

A rigid mass dropped onto a pure linear spring will experience a half sine acceleration pulse as the spring compresses and then expands.

The square wave, or, more accurately, rectangular, pulse (Fig. 21b) is simple in concept, but can only be generated approximately. The infinite slopes at time 0 and time τ necessitate instantaneous rise and decay times, and no equipment yet exists that can meet this requirement. The square wave is analytically expressed as:

$$\begin{aligned} a(t) &= A & [0 < t < \tau] \\ a(t) &= 0 & [t > \tau,\ t < 0] \end{aligned} \qquad (29)$$

Successful square wave production normally requires fairly sophisticated equipment, such as shock machines utilizing metered fluid flow for pulse shaping. One important feature of this pulse shape is its rich content of higher frequency harmonics.

The sawtooth pulse (Fig. 21c) is characterized by an acceleration that increases linearly to a maximum level, and then drops abruptly to zero. Again, the instantaneous decrease to zero can only be approached, but the sawtooth form is nevertheless sufficiently valuable as to be rapidly becoming the recommended pulse for many types of test work. The (terminal peak) sawtooth is expressed analytically as:

$$\begin{aligned} a(t) &= \frac{A}{\tau} t & [0 < t < \tau] \\ a(t) &= 0 & [t > \tau,\ t < 0] \end{aligned} \qquad (30)$$

One popular technique for obtaining this pulse shape involves dropping a suitably mounted test specimen onto a deformable pellet, commonly a small cylinder of lead.

Another and different type of shock motion is the decaying sinusoid, or transient vibration, shown in Figure 22. This motion is identical to the free vibration of an under-damped single degree of freedom system, which was discussed earlier. It is expressed analytically as:

$$a(t) = A \exp\left(-\frac{ct}{2m}\right) \sin(\omega t + \phi) \qquad [t > 0] \qquad (31)$$

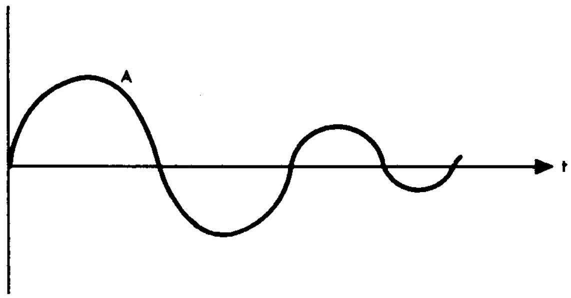


Figure 22. Decaying sinusoid or transient vibration.

(Compare with Eq. 13.) This motion may be considered as the simplest type of shock response. It may be generated by exciting a lightly damped single degree of freedom system (Fig. 8) with a simple impulse.

Complex Shock

The complex shock shown in Figure 23 is typical of actual data obtained in the field. Because of its highly complex nature, a shock motion of this type cannot be described analytically; because of its frequent occurence in practice, it cannot be ignored. As a result, powerful methods have been developed to deal with it. Very large equipment interacts with, or loads a shock machine to which it is attached, with the result that transients generated by the machine produce complex shock. Equipment mounted in vehicles may experience complex shock that is the end result of a shock load—such as an aircraft landing — which is transmitted through the vehicle structure.

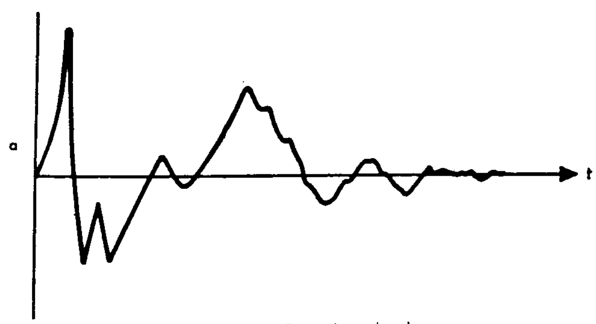


Figure 23. Complex shock.

Fourier Integral

In the discussion of complex vibration, it was pointed out that all periodic waves can be represented as a sum of sinusoidal or simple harmonic waves, and that such a representation is known as a Fourier Series. The various Fourier components occur at discrete frequencies, as shown in Fig. 24, the Fourier spectrum of the complex wave. In a similar way, any non-periodic function may also be represented as a sum of sinusoidal components. In contrast to the periodic case, however, the Fourier spectrum of a non-periodic transient wave is a continuous function (Fig. 25) and is obtained by integration. The Fourier integral of a shock wave is written:

$$F(\omega) = \int_{-\infty}^{+\infty} a(t) \exp(-j\omega t)\, dt \qquad (32)$$

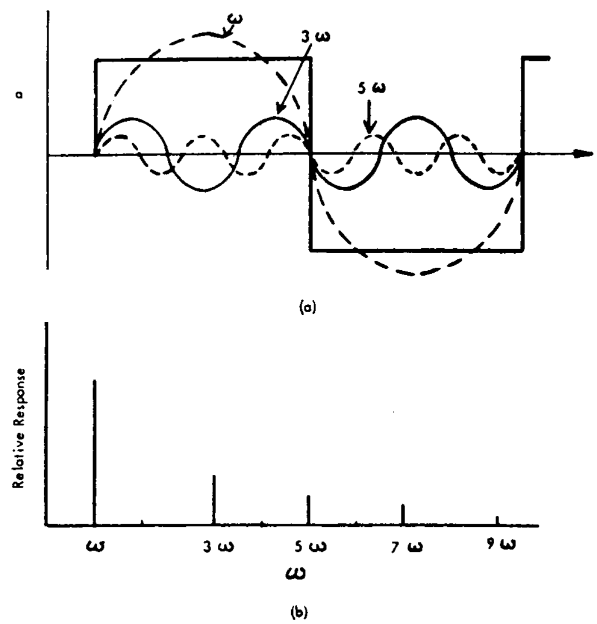


Figure 24. (a) Square wave showing first three sinusoidal components, (b) Fourier spectrum of square wave (periodic).

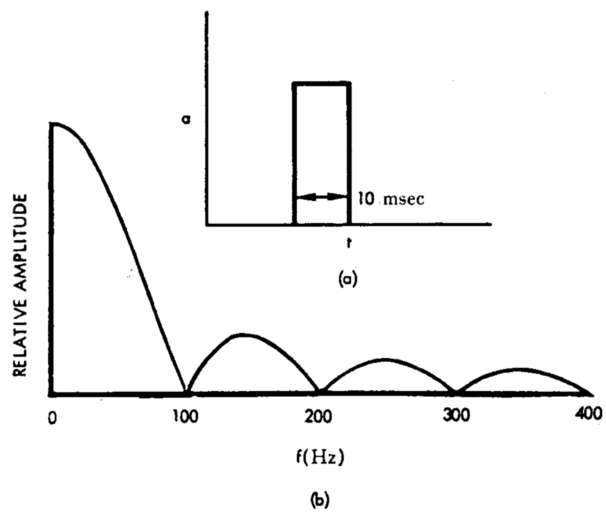


Figure 25. Non-periodic transient: (a) 10 msec rectangular pulse, (b) Fourier spectrum of a 10 msec rectangular pulse.

In addition to its value in data reduction, Fourier spectral information is useful in determining the required frequency response of instrumentation used for shock measurement. For instance, the measurement of shock often requires better low frequency response than might be anticipated from the basic shape of the transient pulse. As an example, it may seem at first glance that a single ten-millisecond half sine, rectangular, or sawtooth pulse contains no frequency components lower than 50 Hz, the frequency corresponding to a ten-millisecond half period and/or a twenty-millisecond full period of a continuous sine wave.

In the case of the continuous and repetitive waveforms shown in Figure 26, there are no frequency components lower than 50 Hz. The repetitive square wave contains higher harmonics in addition to the fundamental 50 Hz frequency. The only frequency component of the sine wave is 50 Hz.

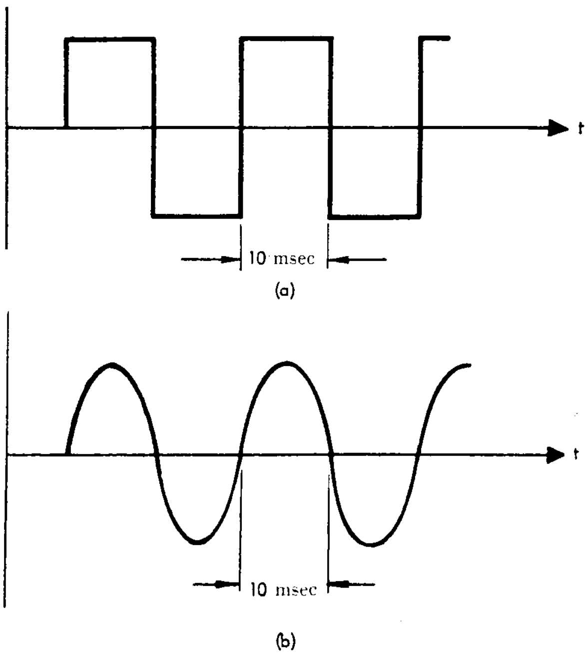


Figure 26. Repetitive wave forms.

If the square pulse is a repetitive transient, a new situation exists. In the case of Figure 27, a 10 millisecond square pulse is repeated at a rate of 10 times per second. In this case the lowest frequency component exclusive of the DC average is 10 Hz, corresponding to the repetition rate of the pulse.

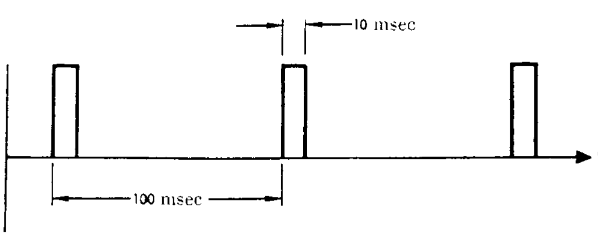


Figure 27. Repetitive rectangular pulse—lowest frequency component 10 Hz

If the repetition rate is slowed and the time between pulses lengthened, the lowest frequency component is correspondingly reduced. With one pulse per second, the lowest frequency is 1 Hz. With one pulse per minute, the lowest frequency is 0.0167 Hz.

A single pulse corresponds to the situation where the time between pulses is lengthened until it becomes infinite; the lowest frequency component then approaches zero Hz. The relative frequency spectrum of a single 10 millisecond rectangular pulse is shown in Figure 25.

If a small amount of distortion can be tolerated, response all the way to zero frequency is unnecessary. However, response below 5 Hz is normally advisable for pulses that exceed several milliseconds in duration. Much of the spectrum of the pulse in Figure 25 will be lost if the measuring system has no response below 50 Hz.

Shock Spectrum

If an undamped linear single degree of freedom system is subjected to some given shock motion input, the resultant motion, or response, of the system will be determined by (1) the amplitude, pulse shape and time duration of the shock input, and (2) the system natural frequency. Response of such a system to various pulse shapes and durations is shown in Figures 28 and 29.

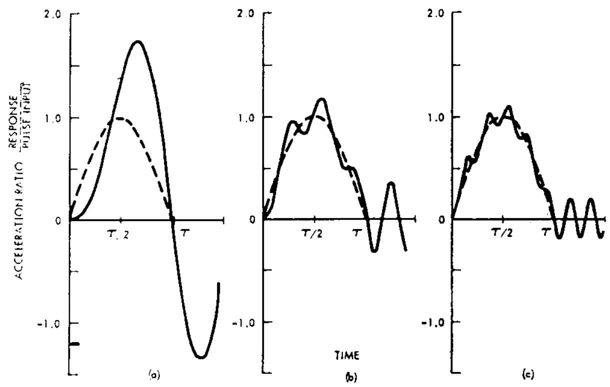


Figure 28. Acceleration response to a half sine acceleration pulse, of duration T, of an undamped single degree of freedom system whose natural period is equal to: (a) 1.014 times the pulse duration, (b) 0.338 times the pulse duration, and (c) 0.203 times the pulse duration (after Levy and Kroll).

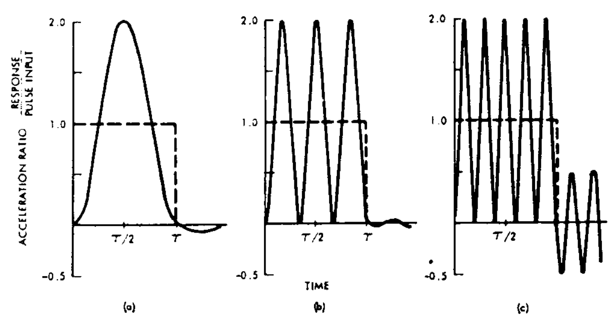


Figure 29. Acceleration response to a rectangular acceleration pulse, of duration T, of an undamped single degree of freedom system whose natural period is equal to: (a) 1.014 times the pulse duration, (b) 0.338 times the pulse duration, and (c) 0.203 times the pulse duration (after Levy and Kroll).

Suppose that several such resonators, each with a different natural frequency, are mounted to a rigid structure and subjected to the same shock input. If the maximum response of each system is recorded and plotted as a function of resonant frequency, the result is the response spectrum for that shock input. In practice, a continuous spectrum—corresponding to a very large number of resonators differing only slightly from one another in natural frequency—is normally plotted.

In particular, a shock spectrum is usually defined as the maximum acceleration responses of a series of simple systems to the shock motion (Fig. 30).

Examination of Figures 28 and 29 shows that the response of a system during application of a shock may differ considerably from the residual motion after the shock input has ended. Because of this, it is customary to consider the primary spectrum (response during shock input) and the residual spectrum (response after shock input) separately. The primary spectrum defines a response that always has the same direction as the applied acceleration, while the residual spectrum defines peak acceleration in both directions. The residual spectrum is important not only in providing more information about dynamic loading, but because it also indicates the fatigue loading due to flexural motion. In view of this, it can be seen that a shock test that provides a flat response in both the primary and residual spectra would be highly desirable. Such a test would be equally severe in the loading it imposes on all equipments tested, regardless of their natural frequencies. Further, a spectrum that rises smoothly to about 100 Hz and is flat for all higher frequencies is a very good average of the shocks actually encountered in transporting and handling equipment. A terminal peak sawtooth acceleration pulse exhibits a spectrum of this type. In addition, the primary and residual spectra are almost equal (Fig. 30b).

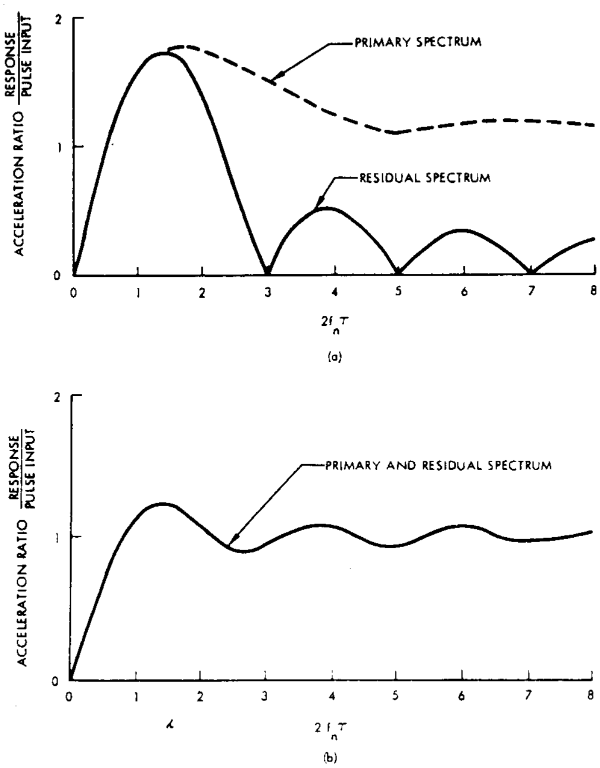


Figure 30. Shock spectrum of: (a) a half sine pulse of period T, and (b) a terminal peak sawtooth of period T (after Lowe).

Shock spectra can be used to compare intensities of different shocks and are directly applicable to several dynamic design techniques, and the numbers quoted correlate better with the damage potential of the related shock. The difficult job of placing tolerances on pulse shapes is avoided and tests can be conducted that compare very well with actual field conditions. One of its great values is that it permits useful analysis of complex shock. It should be remembered that the shock spectrum tells what a shock does, not what it is.