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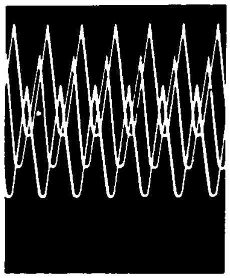

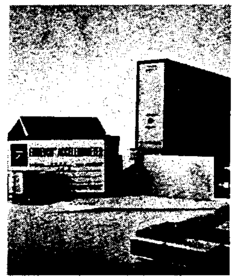

TP 202

MECHANICAL IMPEDANCE TESTING

By Wilson Bradley, Jr., Endevco Corporation

INTRODUCTION

Mechanical impedance at a point on an object is the ratio of the applied sinusoidal force required to achieve a specified motion response at that point. Laboratory vibration testing of an object usually assumes that the vibration environment is defined when the displacement or acceleration spectrum versus frequency of the environment at its mounting location is known. This is not often true. The object under test influences its own environment in a way which is dependent on the ratio of its mechanical impedance to the mechanical impedance of the driving or mounting structure at each frequency. Examples show extreme effects when mechanical impedances are ignored.

Mechanical impedance data must then be known for both the driving structure and in some cases the object under test. Impedances of structures are readily measurable with small portable shakers and impedance measuring gages or equivalent that provide accurate force, acceleration, and phase measurement at each frequency of interest.

With impedance data and knowing the vibration level of the unloaded driving structure, the vibration environment is truly defined and performance can be predicted with improved reliability.

MECHANICAL IMPEDANCE

Mechanical impedance (Z) at any one frequency (ω) is defined by the following equations:

Where: x = sinusoidal displacement
$\dot{x}$ = sinusoidal velocity
$\ddot{x}$ = sinusoidal acceleration
ω = radial frequency ($2\pi f$)
F = driving sinusoidal force

$$Z = \frac{F}{\dot{x}} \qquad (1)$$

or

$$Z = \frac{F}{x\,\omega} \qquad (2)$$

or

$$Z = \omega\frac{F}{\ddot{x}} \qquad (3)$$

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As indicated by Equation (1), impedance is the magnitude of the force F applied to a single point of a structure divided by the velocity ($\dot{x}$) which results from the applied force. All terms are vectors and the phase angle ϕ between the applied force and resultant velocity must also be known to completely define the impedance of that specific point in the structure.

The fact that mechanical impedance of almost any object is frequency dependent is illustrated in Figure 1. Curve (a) describes the point impedance of an idealized mass which increases in impedance with increasing frequency. Curve (b) describes an idealized viscous dash pot or damper. Curve (c) describes an idealized spring which decreases in impedance with frequency. The fact that all structures are complex combinations of these three idealized components leads to typical point impedance graphs as shown in Figure 2. It is these wide variations of more than 100 to 1 in impedance values that make it necessary to consider the effect of structural impedances on shock and vibration reliability.

The following examples illustrate the importance of impedance data:

Example I. Consider the problem of mounting rotating equipment such as turbines, motors, or generators to the structural frame of a building, ship, aircraft, or missile. The rotating device, no matter how accurately balanced, will produce at its mounts a small vibratory force (F) at a frequency corresponding to its rotating frequency (f_r). Let the mechanical impedance of the building frame be as shown in Figure 2. It is shown expanded at the frequency of interest in Figure 3. The exact forcing frequency (f_r) could be anywhere but consider two possibilities at f_a and f_b. If f_r corresponds to f_a, a small amount of vibrating motion will result because

$$\dot{x}_a = \frac{F}{Z_a}$$

while a rotating frequency corresponding to f_b will cause motion 100 times larger that may result in

1) failure of the structure,
2) excessive structureborne, airborne, or waterborne noise.

For best results, then, it is necessary to know the following:

a) The unbalance forces at f_r and any secondary frequencies.
b) The impedance of the structure (with equipment mounted) in the region of these frequencies.

c) Resultant motion calculations using (a) and (b) above determine whether performance is acceptable. If not acceptable, then the following opportunities are still available:

1. Vibratory forces can be reduced by better balancing.
2. Structural impedances can be raised at critical frequencies by stiffening, damping, et cetera.
3. An isolation mount can be placed between the rotating equipment and structure to change the impedance of the system.

Example II. Consider the problem of design and reliability testing of electronic equipment that must be carried in a missile whose idealized vibration characteristics (Fig. 4a) have been predicted, based upon (1) engine vibration characteristics, (2) structural resonances, and (3) aerodynamic and acoustic excitation. The environmental test (Fig. 4b) derived for the electronic equipment to be mounted in the missile usually conservatively blankets the peak values in the predicted environment. At this point the tacit assumption has been made that the structural impedance is many times higher than the mechanical impedance of the equipment. This assumption can lead to over-testing by large factors because the structure at its predicted peak acceleration values is at resonance, which, by definition, corresponds to very low impedance values (i.e., at resonance a structure requires almost no force to create large vibratory motions). Therefore, in many instances the equipment impedance exceeds the structural impedance at these frequencies, causing a drastic reduction in the peak vibratory motion (Fig. 4c) when it is mounted in place by a factor related to the ratio of the equipment impedance (Z_e) to structural impedance (Z_s) as shown below:

$$\dot{x} = \frac{F}{Z_s} \qquad (4)$$

$$\dot{x}_1 = \frac{F}{Z} \qquad (5)$$

$$Z = Z_s + Z_e \qquad (6)$$

From (5) and (6):

$$\dot{x}_1 = \frac{F}{Z_s + Z_e} \qquad (7)$$

(Note that $Z = Z_s + Z_e$ is true in this case because the impedances are considered to be in parallel with each other. Series connected impedances such as shock isolators add in reciprocal; i.e., $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \; \cdots\cdots \; \frac{1}{Z_n}$)

From (7) and (4):

$$\dot{x}_1 = \left(\frac{Z_s}{Z_e + Z_s} \right) \dot{x} \qquad (8)$$

$$\text{If } Z_e > Z_s \text{ then } \dot{x}_1 < \dot{x} \qquad (9)$$

where:
Z = combined impedance of Z_s and Z_e
F = predicted driving force
$\dot{x}$ = velocity of vibration unloaded
$\dot{x}_1$ = velocity of vibration with equipment in place

Example III. Example II can lead to under-testing as well. Consider what happens when there exist vibratory forces (F_1) (such as from the engine) much larger than F in Example II at frequencies below the resonant point examined in that example. The addition of the impedance of the equipment to the structure usually lowers the resonant frequency point and may cause it to coincide with a frequency where the larger forces F_1 exist.

Then: $F_1 >> F$ and $\dot{x}_1$ may greatly exceed $\dot{x}$ causing an even more severe environment than that which was predicted ignoring impedance values and the effect of adding other impedances to the system.

DIRECT MEASUREMENT METHODS

The increasing use of the concept of mechanical impedance of structures requires methods of determining the quantity. Analytical methods are available, but this discussion is limited to experimental methods in which the driving forces and the resultant motions may be measured directly.

At any point in a structure, rectilinear displacements are possible in any one of three mutually perpendicular axes. Thus, six values of mechanical impedance would have to be determined to specify completely the impedance at a given point; this would require an "impedance tensor." In most practical cases, however, the impedance in one direction, or occasionally two, is all that is needed. This discussion is confined to direct determination of impedance in one direction only, and will not go into the problem of combining impedances in different directions. This simplification is possible when the direction chosen is along an axis of symmetry.

Mechanical impedance values usually vary widely with frequency. Since the impedance may be of interest from a few cycles per second to 5000 cps, or higher, the values are most clearly expressed graphically, or in tabular form, showing all frequencies of interest.

The quantity Z is a complex quantity, $Z = F_o/\dot{x}$, and requires for its specification either its real and imaginary parts, or its absolute value and phase angle. Hence, Z must be expressed either in the form "a + jb" or in the form "$Ae^{j\phi}$."

To determine mechanical impedance, Z, experimentally, measurements must be made for each frequency of interest. These measurements must include the motion at the point of application, the magnitude of the sinusoidal driving force at that same point, and the phase relation between the two. The following general requirements are thus necessary in order to measure mechanical impedance:

A. A source of vibratory force variable over the desired frequency range.

B. Method(s) of measuring the vibratory force applied to the specimen.

C. Method(s) of measuring the resultant motion of the specimen due to the driving force.

D. Method(s) of measuring the phase angle between velocity and force vectors.

E. In place of B and C a single instrument to measure force and vibration.

F. Method(s) for plotting and recording data versus frequency.

A. Force Generation

The vibratory force required for a specific mechanical impedance test has definite upper and lower limits. It must not be so large as to damage the specimen or cause it to operate under non-linear conditions; but the force must be large enough to produce an accurately measurable motion. For example, assume that a mechanical impedance measurement is desired on a large structure at 500 cps. If the effective mass of the specimen at 500 cps is estimated at approximately 1000 pounds, then a minimum driving force of 100 pounds would be required to produce a measurable acceleration of 0.1 g ($F = M_a$). In most cases, however, effective mass values are considerably smaller and vibratory forces in the range of 1 to 10 pounds are sufficient for mechanical impedance measurements.

The electrodynamic vibration generator, or shaker, is useful over a wide frequency range -- from 5 cps to as high as 10,000 cps. When the shaker is used in a laboratory, it is usually mounted to a large mass which absorbs the reaction forces. If it is to be used in the field, where a massive mounting structure is impractical or impossible, a reaction mass is needed.

This mass will raise the impedance of the shaker system so that greater testing forces are available. The vibration generator and reaction mass are usually suspended from cables at the desired angle for attaching to the specimen.

B. Force Measurement

At low frequencies many sources of vibratory force permit direct calculation of force measurements, when it can be proven that the material between the force generator and the point of force application to the specimen is reacting as an ideal spring without damping and well below resonance. An eccentric mass vibration generator allows direct computation of the force input into the specimen, up to several hundred cps. Similarly the force input provided by an electrodynamic shaker can be determined by measuring driving current into the vibration generator's moving element. This measurement may be accurate up to at least 500 cps, providing that there is minimal material between the driving coil and the point of force application to the specimen.

In most instances, however, the forces are measured more accurately with separate instruments. A gage is placed between the vibration generator and the specimen under test, as closely as possible to the specimen. The end of the gage between sensing material and specimen must be stiff enough to transmit the applied force to the specimen without resonance (caused by the mass of the specimen and the spring constant of the end of the gage) occurring over the frequency range of interest.

A crystal force gage (Fig. 5) can be used, providing very high sensitivities and frequency response over the entire spectrum from 2 cps to 5000 cps. When large forces are encountered, lower sensitivity bonded strain gages can be used.

C. Motion Measurement

It is not necessary to measure the sinusoidal velocity directly. The impedance may be derived from measurements either of displacement or of acceleration. At low frequencies velocity and acceleration have very small values, so displacement is the most easily measured. Up to 100 cps displacement may be measured with a macroscope. A stroboscopic light will improve accuracy. In general, however, a seismic type of transducer utilizing potentiometer, differential transformer or capacitance bridge principles is used. Most of these transducers are suitable for use up to several hundred cps.

Over a middle frequency range velocity may be measured directly with a "velocity pickup." This unit is a self-generating seismic transducer providing a voltage directly proportional to velocity. When point positioning is required, the size and weight of the unit may be a drawback. Accurate measurement by a velocity pickup is limited to an upper range of about 1000 cps.

Crystal type accelerometers measure motion effectively over the frequency range of 2 cps to 10,000 cps. Their small size makes it possible to place them in close juxtaposition with the force measuring instrument (Fig. 6). At high frequencies it is imperative to measure driving force and resultant acceleration at the same point to avoid the large errors introduced by decoupling between two separate points. The crystal type accelerometer has the further advantage of not using damping, and therefore not introducing phase shift. The other transducers mentioned above may be operated without damping or phase shift, but this limits the frequency range.

D. Phase Measurement

When a rotating eccentric source of vibratory force is used, phase measurements can be made very accurately by use of an interrupter actuated by the mechanical generator at a specific point in its cycle. This interruption can be transmitted to a recorder which monitors motion as well, indicating the phase relationship of the motion as compared to the forcing function.

For other systems the phase angle between the velocity and force vectors should be precisely measured with a phase meter. This measurement must be accurate within $\pm 1^{\circ}$ when either real or imaginary components of the mechanical impedance are small. It is important to realize that in many instances true phase relationships can be lost unless the following precautions are observed:

1) Both transducers must have either zero phase shift or linear phase shift over the frequency range. Transducers free of damping are thus preferable. Damping designed to provide linear phase shift usually does so at only one temperature.

2) Both transducers must act effectively at the same point so that the impedance of the point is measured rather than the transfer impedance between two points.

3) When differentiation or integration to velocity must be made, it is important to preserve the true phase characteristics. This suggests the advantage of a graphical method in which only simple 90° phase corrections are required.

4) Many phase meters lose their accuracy when sinusoidal signals containing noise or harmonic distortion are encountered. It may be necessary to select frequencies where simple harmonic motion exists, to use narrow band filtering, or to use special phase meters designed for this purpose.

E. Impedance "Heads"

An impedance head is a single instrument which contains both the force sensing and motion sensing transducers. It can have advantage over separate instrumentation:

1) Theoretically both force and motion can be measured at the same point, eliminating motion decoupling and phase shift errors.

2) Provides smaller size and simpler mounting than separate instrumentation.

Figure 7 shows such an instrument schematically where both force and acceleration are measured and averaged over the same circle C_{Fa} satisfying advantage 1) above. The mounting configuration is also shown with provision for mounting a small electrodynamic vibrator as an integral part of the system or separately suspending a larger force generator. Figure 8 is a commercial impedance head, Endevco Model 2110. Its sensitivities are nominally 5 mv per pound of force and 70 mv per g acceleration. The high frequency limit is usually determined by a stiffness line which, for this gage, is approximately 2" in diameter with clearance for a 1/2" mounting bolt through the center and is approximately 1" thick. The center bolt mounting with the adapters approaches as nearly as possible the configuration for measuring the true impedance at a single point on the specimen, without introducing the distortions of localized impedances caused by multi-point mounts.

F. Calculations and Plotting

Once the driving force, the resultant motion and the phase angle between them are known with accuracy, the results are calculated and plotted. One of the more useful techniques is to use the graph paper of Figure 9, which is shown with a typical mechanical impedance plot. This graphical technique is of particular advantage when the motion has been measured as a function of acceleration, because no conversion to velocity is required before plotting.

Most mechanical impedance measurements are made in terms of acceleration. The formula $F = M_a$ applies, from which is derived $M = F/\ddot{x}$. In this instance the measured force divided by the measured acceleration, $\ddot{x}$, yields the measured effective mass of the system. Again a plot can be made directly on the graph paper of Figure 9 at the intersection of the effective mass line and the frequency line. For greatest accuracy it is necessary to subtract from the measured mass, M_m, the effective mass, M_o, of the part of the measuring system between the sensing material and the specimen. The effective mass of the specimen, M_s, then becomes:

$$M_s = M_m - M_o$$

This equation results from a simplification of Norton's theorem using two assumptions. First, it is necessary for the impedance head and connecting fixture as seen by the structure to have equal point impedance and transfer impedance. This equality is proven by performing calibrations that demonstrate the impedance head has flat frequency response over the range of intended use. The second assumption is that the motion at the point of attachment to the structure is identical to the motion of the impedance head accelerometers. This assumption is satisfied by carefully designing the fixture so that it is rigid over the frequency range.

Phase angle ϕ, which is the phase angle of the velocity signal behind the force signal, can be plotted as shown on Figure 10. When acceleration is measured, it is important to note that $\phi = \phi_a + 90°$ where ϕ_a is the measured phase angle of acceleration behind the force signal.

The complete mechanical impedance of the specimen at each frequency can then be defined as:

$$Z_s = Ae^{j\phi}$$

where A is the absolute value of mechanical impedance taken from Figure 9 for any frequency. To record the real and imaginary components of the mechanical impedance at each frequency, it is conventional to write in complex notation:

$$Z_s = A \cos \phi + jA \sin \phi$$

REFERENCES

Please see Endevco "Bibliography of Literature on Mechanical Impedance" attached.

CAPTIONS

Figure 1. Impedance characteristics of (a) ideal mass, m, (b) ideal damper, c, and (c) ideal spring of spring constant k.

Figure 2. Typical impedance values for a simple structure.

Figure 3. Range of impedance values between two frequencies f_a and f_b.

Figure 4. Missile environment (a) as predicted, (b) resulting environmental specification for equipment, and (c) actual environment when equipment included.

Figure 5. Crystal force gage.

Figure 6. Impedance tests using separate force and acceleration measuring instruments.

Figure 7. Impedance head and test arrangement.

Figure 8. Impedance head.

Figure 9. Mechanical impedance graph.

Figure 10. Phase angle graph.

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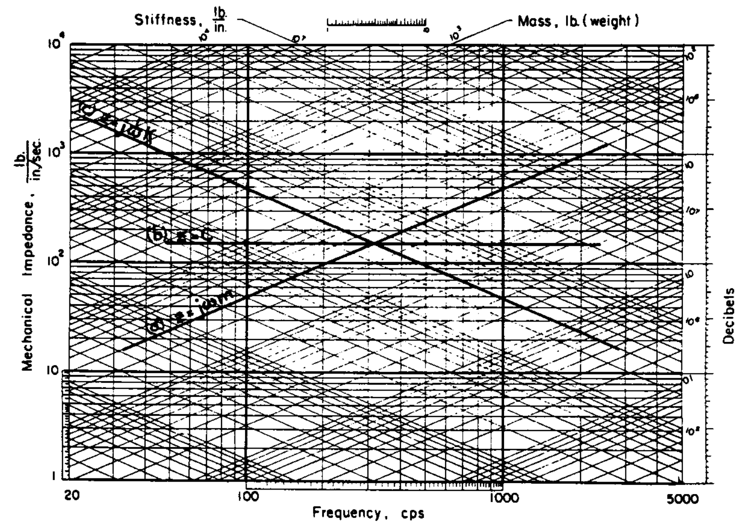


Figure 1

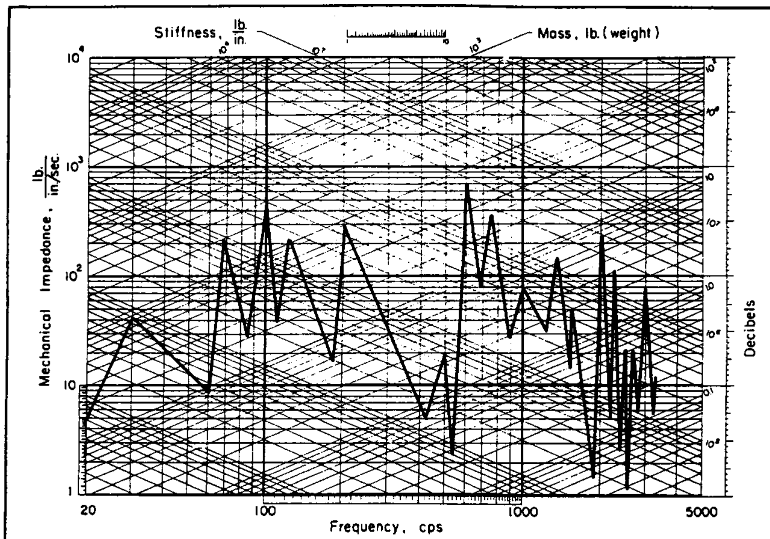


Figure 2

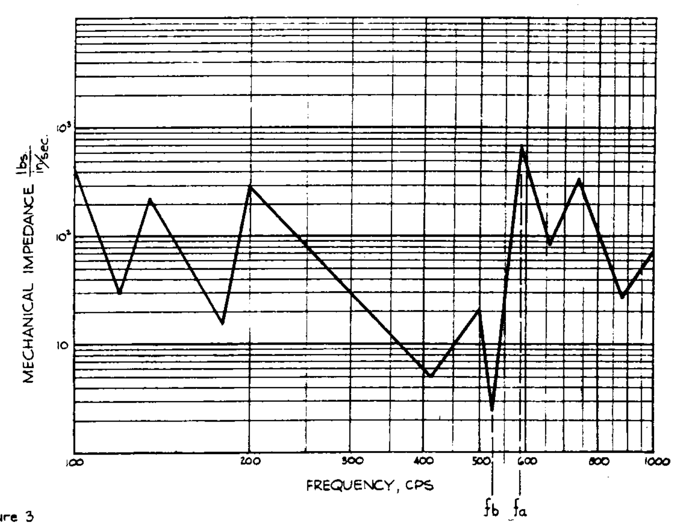


Figure 3

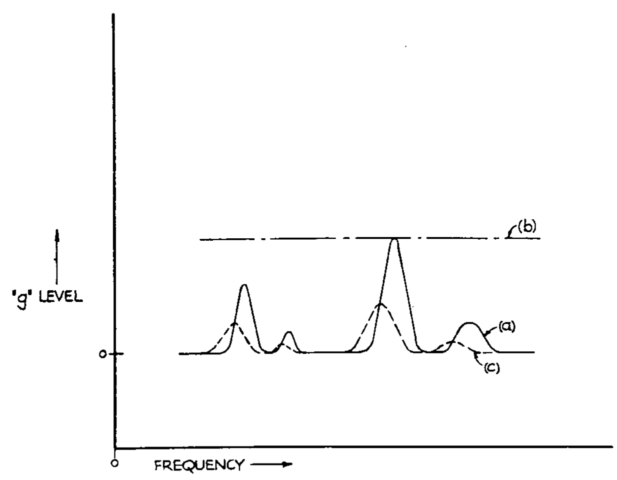


Figure 4

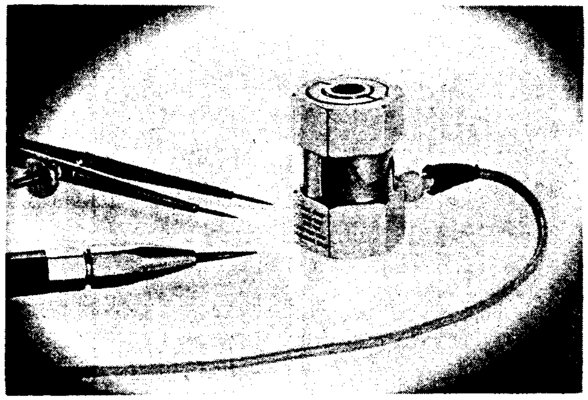


Figure 5

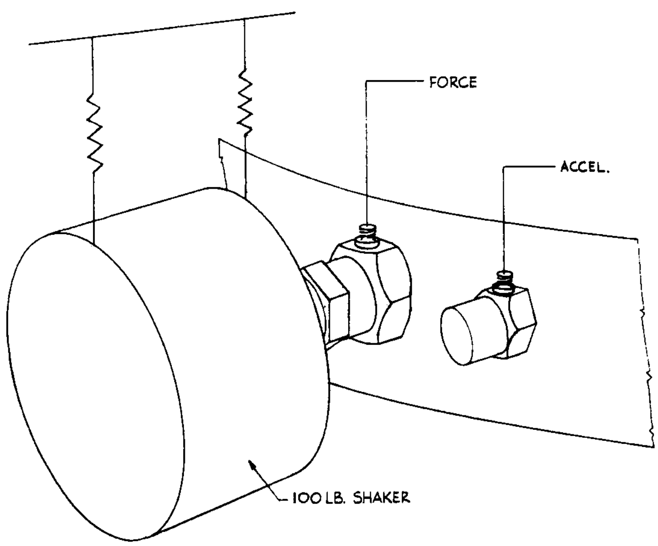


Figure 6

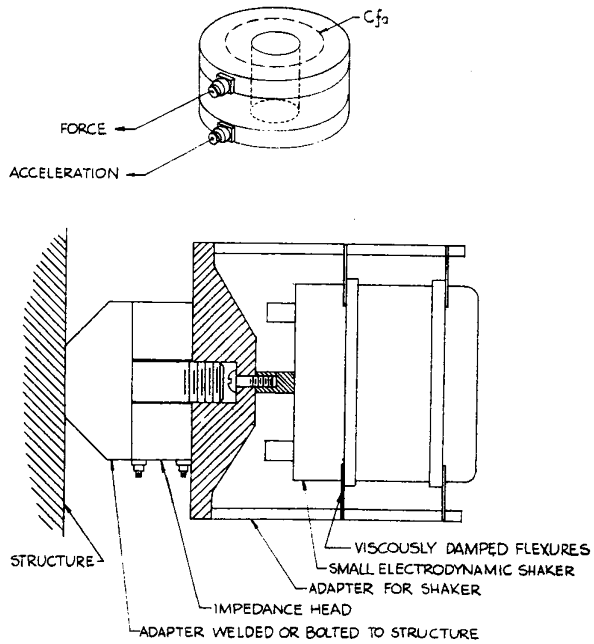


Figure 7

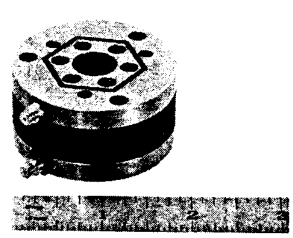


Figure 8

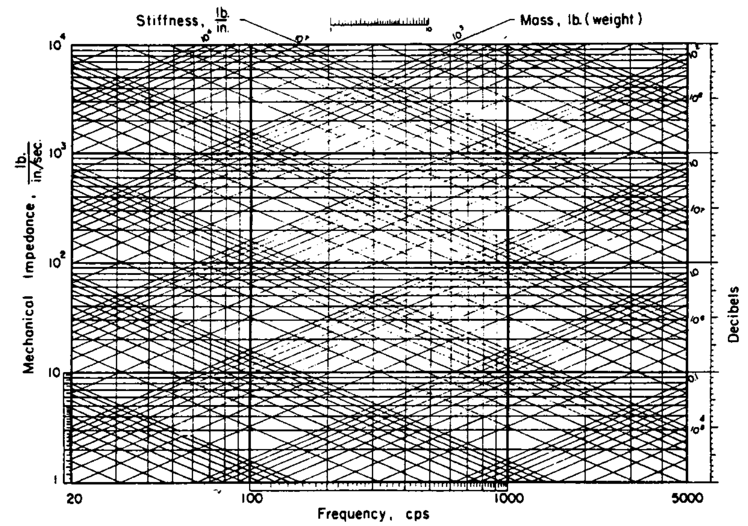


Figure 9

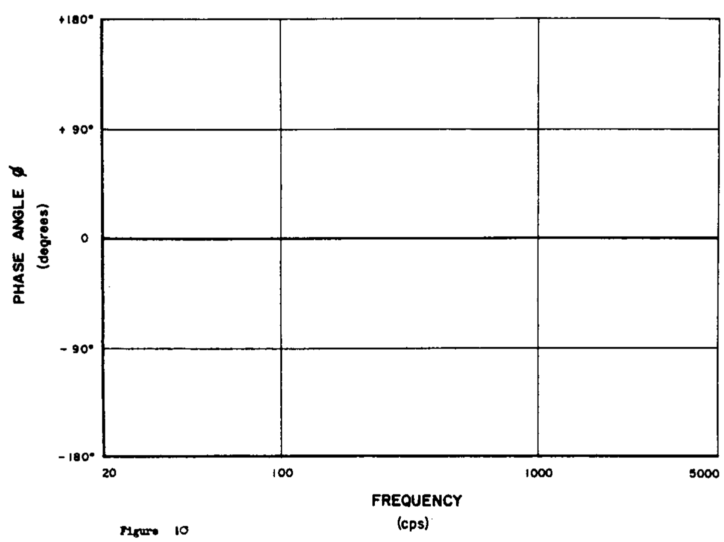
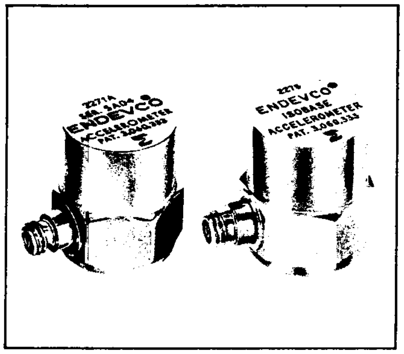


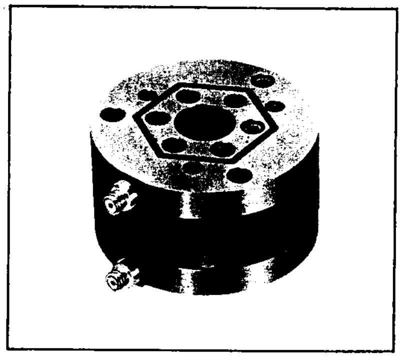
Figure 10

ENDEVCO PRODUCTS RELATED TO THIS ARTICLE



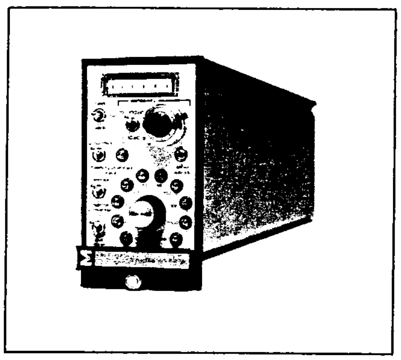
Model 2271A and 2275
Precision Isobase®
Accelerometers

The Models 2271A and 2275 piezoelectric accelerometers feature extremely low output sensitivity to strain or the bending of their mounting surface. Their wide and useful dynamic characteristics of 2 Hz to 5500 Hz and 0 to 10,000 g permit them to be used for most shock and vibration measurements. Transverse sensitivity is very low, 2% maximum. Flat charge-temperature response, ±3% nominal, over the range of —300°F. to +500°F., is achieved with Piezite® Type P-10 crystal material.



Impedance Head
Model 2110E

Combining an accelerator and a force gage in one case, the Model 2110E Impedance Head simplifies the measurement of mechanical impedance. Its wide frequency response, 2Hz to 5000 Hz, and wide dynamic range, to 500 g and 5000 lbs., makes it useful in almost any application. Center hole mounting by means of a ½ inch stud simplifies attachment. There is no phase shift between acceleration and force output over the useful frequency range.



Model 2730
Charge Amplifier

Designed for use in complex and varied vibration measurement applications, the Model 2730 provides a versatile signal conditioner for piezoelectric transducers. Individual lamps indicate the selected full scale range. Options include calibration oscillator, dc output, galvanometer output, servo output and meter readout. System may be operated with grounded or underground transducers. High gain allows full scale output (10 V pk) for 0.1 g pk input (10 to 100 pC/g sensitivity). Provision is made for insertion of test signal in series with transducer cable. Remote charge converter option is available for long input lines.