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Accelerometer Calibration with Reciprocity Vibration Standards

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ACCELEROMETER CALIBRATION WITH RECIPROCITY VIBRATION STANDARDS

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The first reciprocity-calibrated electrodynamic velocity transducers were separately mounted but mechanically connected to conventional vibration shakers[2, 3]. Later velocity standards were actually built into electrodynamic shakers for reciprocity calibration at low frequencies. Further research made this type of calibration possible also at high frequencies near and above shaker resonances[4]. Primary vibration standards have been calibrated by this method at the National Bureau of Standards for the past decade. This article deals with the reciprocity calibration of piezoelectric accelerometer standards mounted temporarily on an electrodynamic shaker. At frequencies below the lowest axial resonance of the shaker a simplified reciprocity procedure is used. At higher frequencies, the constancy of the standard's sensitivity is verified by comparison with another standard previously calibrated at NBS.

Either the piezoelectric or electrodynamic transducer can be reciprocity calibrated and used for performing comparison calibrations on test accelerometers. However, the piezoelectric vibration standard has two outstanding advantages over the electrodynamic transducer – (1) unique portability and (2) the piezoelectric standard is used up to 10,000 Hz (important for shock motion measurements) while the electrodynamic standard is generally limited to 2000 Hz. A commercially available piezoelectric accelerometer standard is shown in Figure 1 (Endevco Model 2270).

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RECIPROCITY THEORY

The reciprocity theory[4] is applicable to the calibration of vibration standards in the amplitude range where the standard's electrical output is linearly proportional to the shaker's motion. The theory shows a reciprocity relation for the driver coil and equates the ratios of force/current and voltage/velocity where the electrical terms refer to the shaker's driver coil and the mechanical terms reference the shaker mounting table. (The "voltage" is measured when the driving coil is open circuit.) In other words, the driver coil acts as the "reciprocal" transducer whose electrical measurements are used for calibrating the output of the accelerometer standard attached to the mounting table.

RECIPROCITY CALIBRATION EQUATIONS

The derivations and ramifications of the reciprocity theory are developed in References 1, 4. The intent of the present article, however, is to detail the calibration procedure itself. For this purpose, only the necessary equations will be given, as follows:

$$S_a = 2635\sqrt{JR/jf} \quad (1)$$

where

S_a = sensitivity (mV/g) of the piezoelectric accelerometer standard

J = transfer admittance intercept (the intercept of a plot of weight/transfer admittance ratio, lb/mho, where transfer admittance ratio is driver-coil current/piezoelectric-accelerometer voltage output, and weight refers to the ten 0.1 to 1 lb values used for the calibration here described).

R = voltage ratio: voltage output of the piezoelectric accelerometer standard divided by the voltage output of the driver coil when the shaker is driven by a second driver coil and the first driver coil is open circuit.

j = $\sqrt{-1}$, the 90-degree vector

f = the frequency, Hz, at which the voltage ratio is measured

$$J = 0.04204\,\Sigma n^2 Y_{nr} - 0.0502\Sigma\, n^2 W_n Y_{nr} \quad (2)$$

where

n = integers from 1 to 10 corresponding to the number of the weight increment W_n (see Table 1)

W_n = weight increment, lb, from 0.1 to 1.0 in this calibration (also see Table 1)

$$Y_{nr} = W_n/(G_n - G_0) \quad (3)$$

where

G_n = the transfer admittance ratio, as defined for Eq. 1

G_0 = the transfer admittance ratio with no weight attached to the shaker.

Two assumptions made in the derivations of the above equations are valid for most commercial shakers because the calibration is performed at 100 Hz or less, which is small compared with the shaker's axial resonance frequency. The first assumption (in Eq. 1) is that the phase angle of the voltage ratio is 90 degrees. The second assumption (in Eq. 2 and 3) supposes no phase angle between the driver coil current and the output of the piezoelectric accelerometer being calibrated.

The weight increments selected should be in steps of about one-fifth the total weight of the shaker moving parts. For the calibration to be described, ten weights will be used, one pound being the largest.

RECIPROCITY PROCEDURE

The calibration procedure (to be illustrated in detail later by an actual example) can be summarized in the following steps:

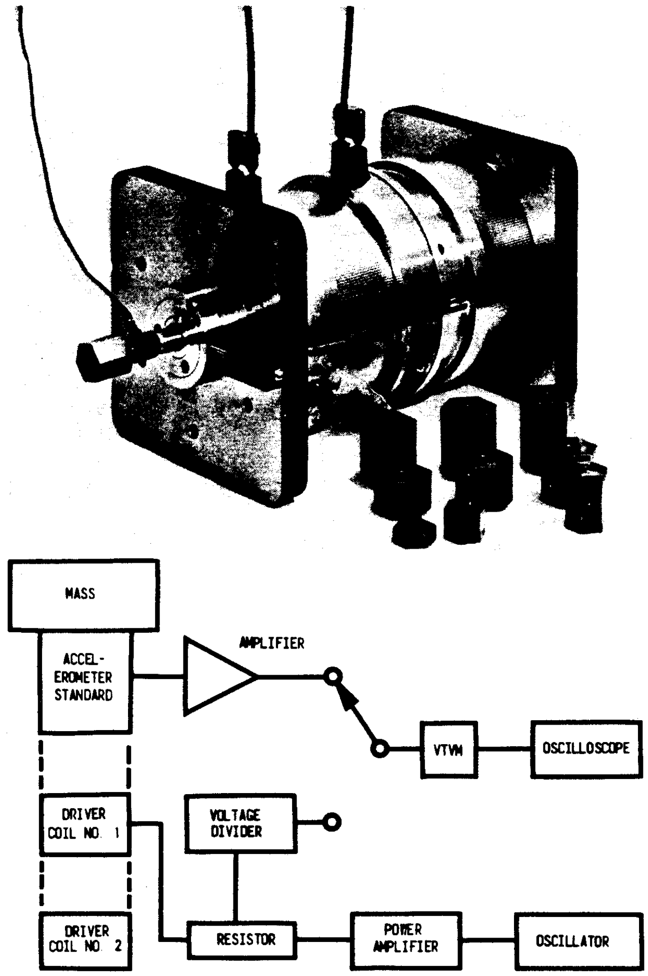

FIGURE 1. Setup used for transfer admittance measurements. Shaker with accelerometer standard and mass attached is shown at top.

1. Calibrate ten weights in equal increments from 0.1 to 1.0 pound on a scale balance.
2. Determine the transfer admittance ratio with each weight and with no weight attached to the shaker. This requires measuring the driver coil current and piezoelectric accelerometer voltage output at each step since these quantities form this ratio.
3. Sum the transfer admittance measurements as indicated by Eq. 2 and 3.
4. Measure the voltage ratio: the piezoelectric standard output/open-circuit driver coil output (when the shaker is driven by a second driver coil or shaker).
5. Use an electronic counter to measure the frequency at which Step 4 is performed.
6. Using the values of J, R, and f from Steps 3, 4, and 5, respectively, compute from Eq. 1 the sensitivity of the piezoelectric accelerometer standard.

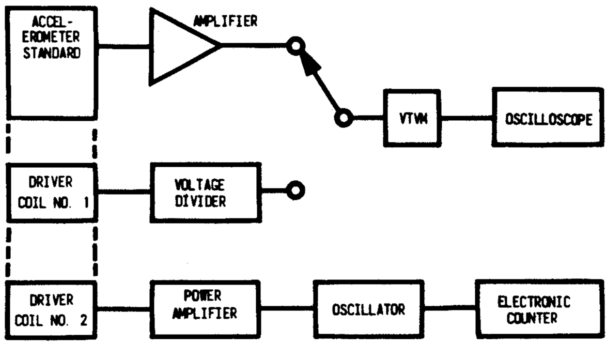

FIGURE 2. Block diagram for voltage ratio measurements.

TABLE 1 – TRANSFER ADMITTANCE MEASUREMENTS ON PIEZOELECTRIC STANDARD PX-SP46 SERIAL NUMBER 10

n	W_n	G_n	Y_{nr}
	lb	amp/ volt	lb-volt/ amp
----	0.0	24.84	----
1	0.1	31.55	.01492
2	0.2	38.17	.01500
3	0.3	44.74	.01508
4	0.4	51.55	.01498
5	0.5	58.14	.01502
6	0.6	64.72	.01505
7	0.7	71.17	.01511
8	0.8	78.03	.01504
9	0.9	84.64	.01505
10	1.0	91.32	.01504
---	0.0	24.84	-----

TRANSFER ADMITTANCE MEASUREMENTS

A sample calibration which follows the above procedure will be described. First to be discussed will be the transfer admittance measurements, the setup and block diagram for which are illustrated in Figure 1. Although two driver coils are permanently built into the shaker, only driver coil No. 1 is used for the transfer admittance measurements.

The shaker selected is free of excessive transverse motion (5) and has a sinusoidal motion whose distortion is less than one percent. Also, the driver coil current and the standard accelerometer outputs have a phase angle of zero degrees as verified by the oscilloscope. All of these shaker characteristics are present throughout an appreciable frequency range including the calibration frequency. The reciprocity calibrations are now performed at 100 Hz to meet industry standards. However, the earlier calibrations summarized in this article were obtained at 50 Hz before the conversion to 100 Hz was made.

Having a satisfactory shaker, each of the ten weights are attached to it for measurements of transfer admittance (G_n) is defined as:

$$G_n = 1/D_n \qquad (4)$$

where D_n is the divider setting which exactly equates divider voltage output to the output from the standard. The voltage divider and resistor, a 1-ohm precision-type, can be General Radio Types 1454A and 500A, respectively.

Table 1 lists the values of G_n and the corresponding values of Y_{nr} computed for a typical reciprocity calibration. When the products of n^2Y_{nr} and $n^2W_nY_{nr}$ are summed and substituted in Eq. 2, J is computed to be 0.01503 1b-volt/ampere for the measurements listed in Table 1.

VOLTAGE RATIO MEASUREMENT

Figure 2 illustrates the block diagram for this measurement. The setup is the same as Figure 1 except the masses are removed and driver coil No. 2 is used to excite the shaker. The oscilloscope is used to indicate that the phase angle of the voltage ratio is 90 degrees, and the electronic counter measures the frequency at which the voltage ratio is taken. As before, an adjustment is made of the voltage divider output until it equals that of the piezoelectric standard.

The voltage ratio, R, then, is:

$$R = D_r$$

where D_r is the setting on the voltage divider. As an example, the value of R was 0.0479 volt/volt at 50 Hz on PX-SP46 piezoelectric standard Serial No. 10.

When Eq. 1 was computed using the values of J, R and f obtained above, a sensitivity of 10.00 millivolts/g resulted for piezoelectric accelerometer standard No. 10. Close agreement with this figure was shown when the same standard was calibrated at the National Bureau of Standards. This latter calibration, an average of 38 points from 10 to 4000 Hz, gave a sensitivity of 10.02 millivolts/g with a 0.5 percent standard deviation.

RECIPROCITY CALIBRATION ACCURACY

The calibration results described above are included in Table 2, which also includes results obtained on other vibration standards calibrated at Endevco by the reciprocity method. All the standards are adjusted so their sensitivities are multiples of 10 for simplicity of use. All are piezoelectric standards except for the first row which covers the electrodynamic standard. In the last column of the table the standard deviation is expressed as a percent of the average sensitivity of all the calibrations performed on each standard.

The piezoelectric accelerometers used in these calibrations are specially designed to have a minimum of calibration errors due to the effects of temperature, strain, and other environmental factors. For these accelerometers, the sensitivity is adjusted prior to the first calibration in Table 2 by inserting capacity or adjusting the gain of the standard amplifier shown in Figure 1 and 2. In the case of the electrodynamic standard the proper value of resistance is put in series with the coil to obtain the desired sensitivity.

An error analysis of the reciprocity calibration method is given in Table 3, where an estimate of the individual errors are listed for the various error sources present. In addition to 100 Hz, the reciprocity errors apply to any frequency at which the 0° and 90° phase angles are present. The square root of the sum of the individual errors squared is determined to obtain the final estimated error.

The 0.5 percent sensitivity error listed in Table 3 agrees closely with the standard deviations for piezoelectric accelerometer standards listed in Table 2. Also, a similar analysis for the electrodynamic velocity standard, reference 1, produces an estimated error of 0.3 percent which is close to the standard deviation in Table 2.

In addition to performing the low-frequency reciprocity calibration for the piezoelectric accelerometer standards, it is necessary to obtain their response at higher frequencies. This is done by a comparison calibration up to 10,000 Hz with an accelerometer standard previously calibrated at NBS. The errors of this comparison calibration are included in Table 3.

The reciprocity and comparison calibration results on a 2270 accelerometer standard are shown in Figure 3. The reciprocity calibration results on 2270 accelerometer standards are included in Table 2. This represents the evolution of reciprocity calibrated standards; the electrodynamic velocity standard, special piezoelectric accelerometer standards and finally the 2270 standard.

TABLE 2 – SUMMARY OF RECIPROCITY CALIBRATION RESULTS ON VIBRATION STANDARDS

Standard	Calibration History, years	Number of Reciprocity Calibrations	Average Sensitivity[a]	Standard Deviation, percent
Electrodynamic[e]	3	10	99.8	0.5
P6SP31#1[e]	1	4	100.2	0.4
P6SP31#2[e]	1	2	99.9	0.1
P10SP46#3	6	10	10.00	0.3
P10SP46#4	6	8	9.97	0.4
P10SP31#5	6	8	100.8	0.8
P10SP46#10[b]	3	3	10.01	0.0
P10SP46#11[e]	2	2	10.05	0.0
P10SP46#12	4	3	10.04	0.3
P10SP46#14[e]	2	3	9.97	0.2
2270 NA09[c, d]	4	13	1001	0.2
2270 NA21[c]	4	8	1000	0.1
2270 PA06[c]	3	4	1003	0.2

a – Units per pC/g or mV/g; for the electrodynamic velocity standard the units are applicable at 50 Hz only.
b – Standard #10 was also calibrated at NBS; 10.02 mV/g was the average sensitivity reported up to 4 kHz with a standard deviation of 0.5 percent.
c – Calibrated with a charge amplifier using the gain range of 1000 mV/g.
d – Standard NA09 was also calibrated from 10 Hz to 10,000 Hz at NBS on the gain range of 100 mV/g; the average of all the calibration points was 100.4 mV/g with a standard deviation of 0.9 percent.
e – Calibrations have been discontinued since these standards are no longer in use.

TABLE 3 – ANALYSIS OF CALIBRATION ERRORS IN DETERMINING THE SENSITIVITY OF THE MODEL 2270 ACCELEROMETER STANDARD AT VARIOUS FREQUENCIES

Reciprocity Calibration 100 Hz

Measurement	Error
	per cent
Mass	0.05
Transfer Admittance Intercept	0.2*
Voltage Ratio	0.2*
Distortion	0.1
Frequency	0.05
Accelerometer Effects, Transverse Sensitivity, Strain, Temperature, etc.	0.2
Amplifier Effects, Gain, Stability, Source Capacitance, etc.	0.3
Estimated Error, at 100 Hz	0.5**

Comparison Calibration 5-10,000 Hz

Measurement	Error
	per cent
Optical Calibration, 5 Hz	1.0
NBS Calibration, 10-900 Hz	1.0
NBS Calibration, 900-10,000 Hz	2.0
Distortion	0.2
Accelerometer Effects, Transverse Sensitivity, Strain, etc.	0.2
Amplifier Effects, Frequency Response, etc.	0.1
Relative Motion, 900-10,000 Hz	0.5
Voltage Ratio	0.2
Estimated Error, 5-900 Hz	1.1**
Estimated Error, 900-10,000 Hz	2.1**

* Assume 0° and 90° phase shifts for transfer admittance and voltage ratio ratio measurements, respectively.
** Determined from the square root of the sum of the squares of the applicable individual errors.

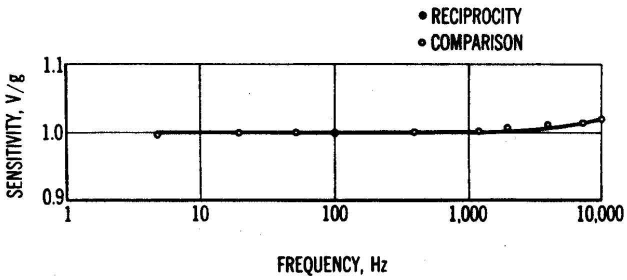

FIGURE 3. Calibration of a Model 2270 accelerometer standard and charge amplifier. The reciprocity calibration performed at 100 Hz and comparison calibration to 10,000 Hz are traceable to the National Bureau of Standards.

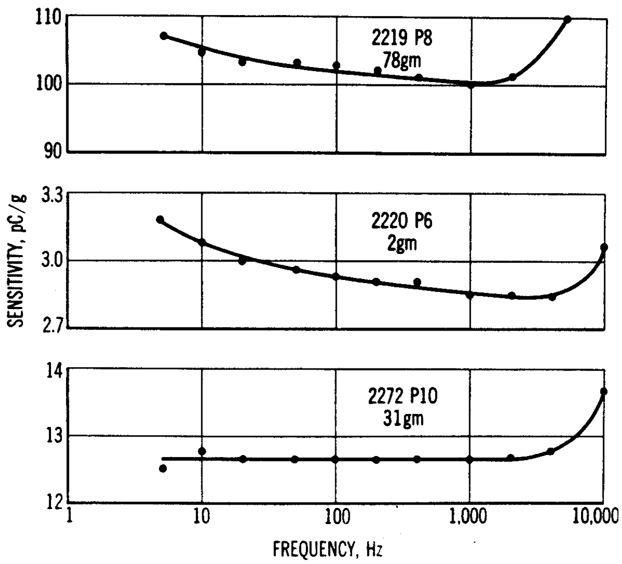

FIGURE 4. Comparison calibration results on three accelerometers obtained with the setup and block diagram illustrated in Figure 4.

SAMPLE COMPARISON CALIBRATION

The reciprocity calibrated standards described above have been used routinely in several calibrators[6]. The 2270 is the most advanced accelerometer standard. It is designed with a case suitable for attaching test accelerometers. This accelerometer standard is reciprocity calibrated and used directly to calibrate single axis piezoelectric accelerometers at frequencies up to 10,000 Hz. The 2270 standard provides special mechanical mounting and electrical grounding features which eliminate non-intentional set-up errors which occur in practice. The 2270 standards are being used to replace the special accelerometer standards in the ca[illegible]ator[illegible] mentioned above[6].

To demonstrate the use of the 2270 standard, a setup and block diagram is used as shown in Fig. 5. As before, the first step here is to adjust the voltage divider output until it equals that of the test accelerometer. The sensitivity of the test accelerometer is then equal to the sensitivity of the standard accelerometer multiplied by the setting on the voltage divider.

Typical calibration results from the above procedure are shown in Fig. 4. The accelerometers selected are representative of the various piezoelectric accelerometers presently in wide use. The weight of these three different accelerometers ranges from 2 to 78 gm, and their sensitivity ranges from approximately 3 to 100 picocoulombs/g. These calibration results indicate the suitability of the piezoelectric standard accelerometer illustrated in Fig. 4 for performing routine comparison calibrations at frequencies up to 10,000 Hz.

COMPARISON CALIBRATION ACCURACY

Table 4 lists an analysis of the errors of the comparison calibration results. The 1.0-percent estimated sensitivity error for the test accelerometer is applicable at the frequency of the previously performed reciprocity calibration. At other frequencies the test accelerometer's estimated sensitivity errors are larger because they include the NBS calibration errors.

The errors listed in Table 4 are typical of those obtained in modern calibration laboratories. At low frequencies calibration differences of less than 2 percent between laboratories are common. Careful selection of accelerometer standards and shakers can maintain the errors near 10,000 Hz at a consistent, though slightly higher value. All this demonstrates that improvement has been achieved over accelerometer calibration errors that existed a decade ago.

CALIBRATION SYSTEM COMMERCIALLY AVAILABLE

The Model 2270 was previously mentioned as a commercially available piezoelectric accelerometer standard. Also available is the System 28350F, which includes the 2270 accelerometer, a Model 2710FM13 amplifier, a Model 2710FM14 amplifier for use with the test accelerometer and a Model 2629A power supply. Although the Model 2270 accelerometer is available for use with any charge or voltage amplifier, this accelerometer was designed for use with Model 2710FM13 charge amplifier and use of the latter is recommended. This standard accelerometer-amplifier system is designed to eliminate errors which frequently occur in calibration laboratories when other standard calibration systems are used.

The System 28350F is supplied with an absolute reciprocity calibration at 100 Hz with an estimated error of 0.5 percent. In addition, calibration from 5 Hz to 10,000 Hz is supplied in a comprehensive report which includes a graph of sensitivity versus frequency. This report supplies all calibration results and shows traceability to the National Bureau of Standards.

SUMMARY

It has been shown that the reciprocity calibration procedure described provides an accurate means for establishing primary vibration standards. The procedure has been simplified by performing the reciprocity calibration at low frequencies below axial resonances in the shaker. In this way the piezoelectric standard's sensitivity is determined within an estimated error of 0.5 percent. Then the sensitivities at higher frequencies can be verified by comparison with a standard previously calibrated at NBS.

Reciprocity calibration is an absolute method for establishing piezoelectric accelerometer standards which should be recalibrated by this method at twelve-month intervals. After being returned to the user's laboratory, these standards are then available for the calibration of other accelerometers by the comparison method. The latter method will limit sensitivity errors to 1 percent at 100 Hz, 1.5 percent up to 900 Hz, and 2.5 percent up to 10,000 Hz.

Recalibration of accelerometer standards can be performed at NBS or at organizations such as the Endevco Corporation, who offer reciprocity calibration on certain accelerometers as a laboratory service along with a variety of other calibration services.

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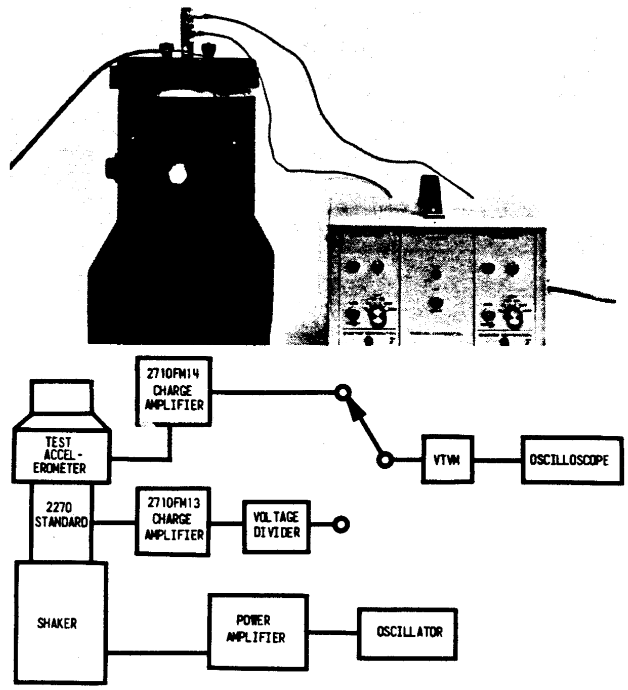

FIGURE 5. Comparison calibration of a test accelerometer using an accelerometer standard previously calibrated by the reciprocity method.

TABLE 4 – ANALYSIS OF ERRORS IN THE SENSITIVITY TEST ACCELEROMETERS CALIBRATED BY THE COMPARISON METHOD

MEASUREMENT	SENSITIVITY ERROR per cent
Reciprocity Calibration Error for Standard, 100 Hz	0.5
Stability of Standard	0.5
Comparison Frequency Response Calibration Error for Standard	
5 Hz - 900 Hz	1.1
900 Hz - 10,000 Hz	2.1
Relative Motion, 900 - 10,000 Hz†	1.0
Distortion	0.2
Voltage Ratio	0.2
Amplitude Linearity – 0.2 g to 100 g	0.2
Range Tracking, Standard Amplifier – 1, 10, and 100 g/V Ranges	0.2
Range Tracking, Test Amplifier	0.2
Amplifier Relative Frequency Response	0.1
Amplifier Gain Stability, Source Capacity, etc.	0.2
Environmental Effects on Accelerometers, Transverse Sensitivity, Strain, Temperature, etc.	0.5*
Environmental Effects on Amplifiers, Residual Noise, etc.	0.2**
Estimated Error – 100 Hz	1.0***
Estimated Error – 5 to 900 Hz	1.5***
Estimated Error – 900 to 10,000 Hz†	2.5***

* The error varies from 0% to 0.5% for most accelerometers operated under controlled laboratory conditions.

** Applies for controlled laboratory conditions.

*** Determined from the square root of the sum of the squares of the applicable individual errors.

† Highest frequency is 5000 Hz for t[illegible] accelerometers with a total mass exceeding 35 grams.

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