Reprinted from THE REVIEW OF SCIENTIFIC INSTRUMENTS, Vol. 36, No. 5, 693–695, May 1965
Printed in U. S. A.

Sensitivity Comparison of Transduction Mechanisms

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(Received 9 September 1964; and in final form, 28 December 1964)

By using as sensitivity the ratio of output power to input mechanical energy, comparisons are made between self-generating (e.g., piezoelectric) and strain gauge type mechanisms. The comparisons indicate the frequencies below which the strain gauge mechanisms are more sensitive than the self-generating mechanisms.

INTRODUCTION

COMPARISONS must be made among available transduction mechanisms in order to select a suitable one for any measurement problem. Because of the great difference in character among mechanisms, the comparisons are often difficult and confusing. The general rules for making comparisons have been outlined in borad terms by Stein[1] and others concerned with transduction theory. This paper presents a generally applicable means of comparing the sensitivity of diverse mechanisms, and demonstrates the technique by comparing piezoelectricity to resistive (and piezoresistive) strain gauges. The approach is not novel; Lee and Pastan[2] used the same terms and concepts to make a limited range of comparisons. What is stressed here is the generality and utility of sensitivity comparisons on the basis of energy in and power out.

SIGNAL POWER

The output of a mechanical-to-electrical transducer is most adequately described in terms of signal power. All transmitting, recording, or display systems, having finite

[1] Peter K. Stein, Instr. Control Systems 37, No. 4, 133 (1964).

[2] S. Y. Lee and H. L. Pastan, Design and Application of Semiconductor Strain Gage Transducers, (ISA Paper 189-LA-61, presented, Sept. 1961, Los Angeles meeting Instrument Society of America). Review in Strain Gauge Readings, IV, No. 5, Dec. 1961, Jan. 1962, Review No. 1381.

input impedance, represent a power drain on the signal from the transducer. Typically, an oscilloscope draws 10^{-10} W of signal power, a 20 μA d'Arsonval meter, 10^{-6} W. The transducer is usually expected to supply this drain without apparent effect on its signal level. Output voltage, which is usually stated to define transducer sensitivity, must be coupled with output impedance to define the useable signal. Using the output signal power, definite relationships to input mechanical signal can be found which characterize various transduction mechanisms.

Theoretically, signals at any voltage and impedance level can be transformed to any other voltage and impedance level at the same power. For ac signals a simple transformer performs this function; for dc signals the transformation is more difficult, but the concept holds true. In most instrumentation work, an impedance mismatch of at least 100:1 is provided between the transducer and the following stage, so the nature of the impedance of the transducer is unimportant. Thus, the dissipation power of voltage across a resistive transducer can be freely compared to the virtual power of alternating voltage on a capacitive or inductive transducer impedance.

MECHANICAL INPUT

Mechanical inputs to transducers tend to be limited in signal energy. Typically, a transducer is a sprung system operated well away from its resonant frequency. The input from the measured system is a force or pressure or acceleration producing an equilibrium distortion (x) of a spring constant ($\mathcal{K}$), or a volume compliance, or the compliance under an inertial mass (or, conversely, a distortion producing a force). In any of these cases, the transducer accepts only an input energy of the nature

$$\text{M.E. (mech. energy)} = \tfrac{1}{2}\mathcal{K}x^2.$$

Unrestrained transducers, such as turbines, which accept unlimited energy from an unchanging mechanical input signal are relatively less common, and will not be treated here.

SELF-GENERATING MECHANISMS

The relationship between signal power and mechanical input is most easily defined for piezoelectricity. For each piezoelectric material and mode of use, there is a coupling coefficient (k) which is the square root of the ratio of electrical energy stored to mechanical energy applied.[3,4]

$$k^2 = \tfrac{1}{2}V^2C/\tfrac{1}{2}Fx = \text{efficiency},$$

where V is voltage, C capacitance, F force, and x deflection. This efficiency is the fraction of mechanical strain energy which appears as electrical stress energy in the piezoelectric material. The rest of the mechanical strain energy is held in the crystal lattice in forms not electrically available. The efficiency of piezoelectric materials ranges from 1% for quartz to 80% for Rochelle salt, with the best lead-zirconate-titanate cermaics running about 50%.

Several assumptions are implicit in the definition of efficiency. The piezoelectric body is assumed to be uniformly stressed, and only the mechanical energy actually put into the piezoelectric is considered. The electrical stress on the piezoelectric is assumed to be uniform, and electrodes applied to equipotential planes in such manner that all the charge is accessible. If these assumptions are met, the efficiency holds regardless of size or shape of piezoelectric body. Cross section and length may be adjusted as required to change stiffness and maximum load; electrodes may be widely spaced to give high voltage or put in layers in the piezoelectric to give low voltage and large capacitance; still the available output energy will be the same fraction of the applied mechanical energy.

The same sort of coupling coefficient and efficiency treatment can be applied to magnetostriction, and efficiencies up to 40% are reported.[5]

The efficiency of a piezoelectric or other self-generating transducer must be multiplied by a time function (frequency) to give signal power per unit energy input. For simplicity, consider a sinusoidal mechanical input with a specified peak mechanical energy:

$$\begin{aligned}
\text{(peak electrical energy)} &= k^2 \text{ (peak M.E.)},\\
\tfrac{1}{2}V_{\text{peak}}^2C &= k^2\text{(peak M.E.)},\\
V_{\text{rms}}^2 &= \tfrac{1}{2}V_{\text{peak}}^2,\\
V_{\text{rms}}^2C &= k^2\text{(peak M.E.)},\\
\text{impedance} &= Z = 1/2\pi fC,\\
\text{virtual power} &= V_{\text{rms}}^2/Z = 2\pi fCV_{\text{rms}}^2,\\
\text{power} &= 2\pi fk^2\text{(peak M.E.)}.
\end{aligned}$$

Thus, the signal power is a constant times the frequency times the mechanical input signal. For a zero frequency (static) input the impedance is infinite and the output power is zero.

NONSELF-GENERATING MECHANISMS

Nonself-generating transduction mechanisms tend to have output characteristics which are invariant with frequency. But, in contrast with the piezoelectrics, the utilization of mechanical energy is a function of the dimensions of the active element. Consider a resistance strain gauge. (The rules would be similar for strain sensitive capacitors and inductors.) The resistance change for the strain gauge is the product of the strain (ϵ) and the gauge factor K,

$$\Delta R/R = K\Delta\ell/\ell = K\epsilon.$$

The energy required to achieve a given level of strain is dependent on the Young's modulus (Y) and the volume of

[3] W. P. Mason, *Piezo-Electric Crystals* (Van Nostrand, Inc., Princeton, New Jersey, 1950).

[4] Clevite Corporation "Modern Piezoelectric Ceramics" Bulletin 9244-1, (1961).

[5] J. Shaw and F. Perry, Electro-Technol. 73, 101 (1964).

material strained,

$$(\text{M.E.}) = \tfrac{1}{2}(\text{vol.})Y\epsilon^2.$$

If the strain gauges are used in a balanced Wheatstone bridge with all four arms active, the unbalance voltage in the bridge will bear the same ratio to the applied voltage as the resistance change bears to the unstressed resistance,

$$V_{\text{out}}/V_{\text{in}} = \Delta R/R.$$

The output power varies linearly with the input power and as the square of the fractional resistance change

$$\text{Power} = V_{\text{out}}^2/R = (\Delta R/R)^2(V_{\text{in}}^2/R).$$

The square of the fractional resistance change is proportional to the mechanical input energy

$$(\Delta R/R)^2 = K^2\epsilon^2 = 2K^2(\text{M.E.})/(\text{vol.})Y.$$

Thus, the output power of a strain gauge bridge is proportional to the input power, input strain energy, and a property of the strain gauge material, and is inversely proportional to the volume of strain gauge material stressed,

$$\text{Power} = 2(V_{\text{in}}^2/R)(K^2/Y)(\text{peak M.E./vol.}).$$

The above is the instantaneous (dc) power. For a sinusoidal mechanical input, the power of the alternating signal is

$$(\text{ac})\text{Power} = (V_{\text{in}}^2/R)(K^2/Y)(\text{peak M.E./vol.}).$$

It is implicit above, that all of the mechanical energy goes into the strain gauges, that the strain level in the gauges is uniform, and that all of the resistance is in the strain gauges. To some extent the permissible input power is determined by the dimensions of the gauges, usually by the area through which heat is conducted away and not by the volume.

COMPARISONS

At sufficiently low frequencies strain gauges and other interrogated transducers give more signal power than self-generating transducers for the same mechanical input. At higher frequencies the self-generating devices tend to give more signal power. Comparisons can be made between specific systems in terms of the frequency at which the power output is equivalent.

As an example, compare a piezoelectric of 50% efficiency with a bridge of silicon strain gauges used unbonded and prestressed so as to accept both tension and compression. For the piezoelectric,

$$\text{Power/peak M.E.} = 2\pi f k^2 = \pi f \text{ W}/J.$$

Probable strain gauge properties are (for a commercial gauge) gauge factor $= K = 120$,

$$Y = 1.9\times10^{12} \text{ dyn/cm}^2,$$

$$\text{vol.} = 4(0.6\times0.0012\times0.05 \text{ cm}) = 0.000144 \text{ cm}^3,$$

input power $= 0.4$ W.

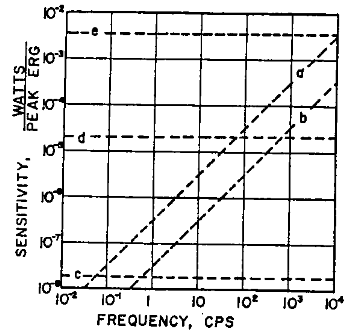


FIG. 1. Sensitivity vs frequency for (a) 50% efficient self-generating mechanism, (b) 5% efficient self-generating mechanism, (c) unbonded wire strain gauge of 4×10^{-6} cm^3, (d) silicon strain gauge, cited in the text, (e) silicon strain gauge of 1.3×10^{-6} cm^3, gauge factor = 150.

For the strain gauges

$$\text{Power/peak M.E.} = 120^2/(1.9\times10^{12})0.4 \text{ W}/(1.44\times10^{-4}) = 2.1\times10^{-5} \text{ W/erg} = 210 \text{ W/J}.$$

The transition frequency, for equal sensitivity, is

$$f = 210/\pi = 67 \text{ cps}.$$

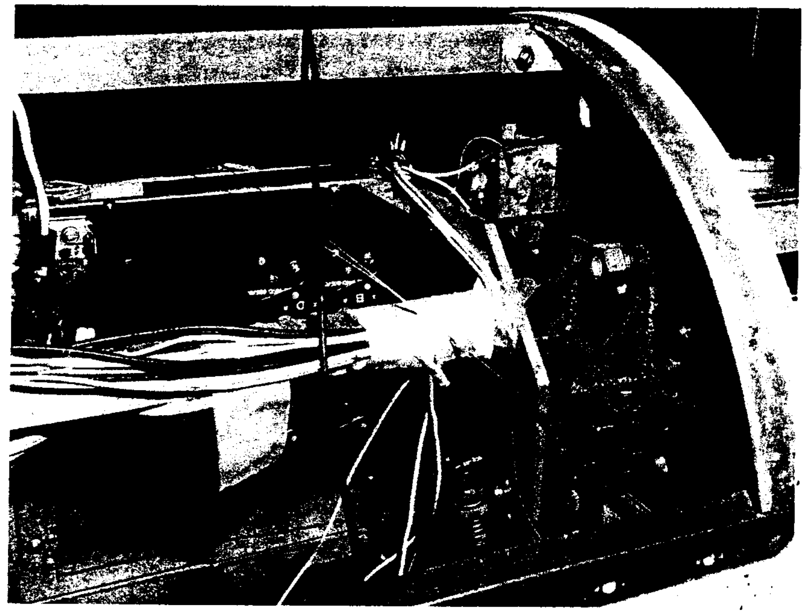
This transition frequency separates the area of high frequency, where the piezoelectric is advantageous, from lower frequencies for which the strain gauge is superior. This frequency is valid only for this piezoelectric and strain gauge set. Other self-generating transducers will have different efficiencies (never exceeding 100%). Other strain gauge systems will have more or less volume strained, may handle more or less power, and may have gauge factors ranging from 1.6 to 175 (or higher).[6] At the present time the transition frequency from the most sensitive strain gauges to the most sensitive piezoelectric is in the 1 kc range.

Figure 1 shows the sensitivity vs frequency relationship of some transducer systems. Most self-generating systems fall between "*a*" and "*b*"; 100% efficiency is only a factor of 2 higher than "*a*", quartz piezoelectrics are only a factor of 5 lower than line "*b*". Line "*c*" represents a commercial unbonded resistance wire strain gauge system. Bonded strain gauges would be much lower. Line "*d*" represents the commercial silicon strain gauge used as an example, above. Most commercial silicon gauges are less sensitive, but some developed specifically for instrument use are more sensitive. Line "*e*" represents a silicon strain gauge being developed especially for applications requiring high sensitivity. Although potentiometer transducers are not "sensitive" in responding to very small inputs, their large power output from appropriate inputs places them at about the level of line "*d*" of Fig. 1.

The signal power-sensitivity concept can be useful in selecting transducers. For similar mechanical input (full scale range and resonant frequency) various transducers will offer various output voltages and impedances. The frequency range of the input can be compared to the input power vs frequency characteristics of the transducers to evaluate useful signals in the pertinent range.

[6] L. Hollander, G. Vick, and T. Diesel, Rev. Sci. Instr. 31, 323 (1960).

calibration method, it is accurate and widely used; therefore, it is also included in the main body of the standard. After a pickup is calibrated by one or more of the above-mentioned standard methods, it may be used to calibrate another pickup simply by comparing the output of the calibrated pickup with that of the pickup being calibrated. Both pickups are mounted to a surface or fixture subjected to the desired constant acceleration, sinusoidal motion, or shock motion. Most frequently, comparison calibrations will be performed on exciters producing sinusoidal motion. To accomplish this, it is necessary to maintain vibration exciters which produce sinusoidal motion free of distortion and transverse motion and to use care to design adequate fixtures.



Three acceleration pickups mounted on the block measure the vertical, horizontal, and lateral motions on this rocket sled thrust plate. Accelerations up to 300g were measured at the igniter pulse during the test. (Courtesy Sandia Corporation)

The two most important methods described in the appendix to the standard are the optical direct viewing and interferometric methods. Direct-viewing calibrations with a microscope can be easily performed with errors as small as ±1 percent. This method is widely used; it is limited, however, to large displacement amplitudes of vibration and is usually used only at frequencies up to about 100 cycles per second. The fringe-disappearance interferometic method is receiving increased recognition because it is being performed with estimated errors of several percent at frequencies up to 10,000 cycles per second and higher. It is usually used to perform calibrations at displacement amplitudes of 4 micro-inches; therefore, it is not used at low frequencies.

In summary, the new American Standard provides information concerning all calibration methods and will be a useful tool to all shock and vibration laboratories. Many laboratories will use the comparison method to calibrate pickups over a wide frequency range. The secondary vibration standard used in the comparison method is calibrated with reference to one of the absolute methods. The National Bureau of Standards recently established a service[2] for calibrating secondary vibration standards.

It is important to calibrate pickups on a regular basis commensurate with their use. After a pickup is subjected to rough handling, after it has undergone severe vibrations and transient motions at high frequencies and accelerations, and before it is used to measure the vibration of important test structures, recalibration should be a matter of routine.

[2] Test Fee Schedules of the National Bureau of Standards—Mechanics, reprinted from *Federal Register*, September 25, 1958, vol 23, No. 188.

Table — Estimated Ranges and Errors of Standard Calibration Methods

Method:	*Tilting Support*	*Centrifuge*	*Rectilinear electrodynamic calibrator*	*Physical pendulum*	*Ballistic pendulum*
Input	constant acceleration	constant acceleration	sinusoidal acceleration, velocity, or displacement	transient acceleration, velocity, or displacement	transient velocity or acceleration
Amplitude	−g to +g	0—60,000g	0—25g 0—50 in./sec 0—0.5 inch	0—10g 0—100 in./sec	2500g
Frequency	0	0	8—2000 cps	0.5 to 5 cps	——
Duration of pulse	——	——	——	——	0.00035—0.001 sec
Maximum weight of pickup	10 lb	100 lb at 100g to 1 lb at 60,000g	2 lb	2 lb	1 lb
Estimated errors of input	±0.0003g	±1%	±1%(8-900cps) ±2%(900—2000 cps)	±2%	±5%