

INSTRUMENTATION FOR SHOCK AND VIBRATION MEASUREMENTS



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Shock and vibration measurements are made on structures, equipment, and high performance vehicles. Instrumentation is available for the accurate measurement of the motion environment present. Only in the past several years calibration methods have been developed which demonstrate that these instruments are capable of accurate measurements at extremes of frequency, acceleration, temperature, etc. This paper describes the performance characteristics of the instruments and the techniques used in shock and vibration measurements including mechanical impedance.

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Nomenclature

- g = Acceleration of gravity, 386 in./sec²
 j = Unit imaginary vector
 v = Velocity, in./sec
 ω = Angular frequency, radians/sec
 w = Weight, lb
 W = Apparent weight, complex ratio of force to acceleration, lb
 Z = Mechanical impedance, complex ratio of force to velocity, lb/in./sec

Introduction

Shock and vibration measurements are made to determine the motions present both in severe environments and in ultraquiet conditions. Certain equipment must function properly when subjected to explosions, oscillatory disturbing forces, and severe acoustic environments. Also, measurements are made in other environments to insure that any motions present are extremely small in order to prevent personal discomfort or detection from without.

For such diversified applications, shock and vibration instruments must be suitable for measuring motions from approximately 0.001 g to 10,000 g and sometimes beyond these limits. It is not practical to design a single instrument system to be suitable for all measurement applications in this acceleration range. On the other hand many systems make accurate measurements throughout a large portion of this range.

The frequency response characteristics for an instrument system is another important requirement. Long duration shock motions require that the response of the system be flat to frequencies sometimes below 1 cps. Other shock motions of extremely short duration are in the microsecond region. These motions and acoustically induced vibrations require that the frequency response be flat to 10,000 cps. The transducer in the system measures the vibration motion produced; but must not have an appreciable output from the high level sound pressures present. This latter characteristic is known as the acoustic response.

Another severe environment encountered is temperature. High and low extremes are encountered in space vehicles, aircraft, etc. Shock and vibration transducers have been developed to operate satisfactorily at temperatures in the range from -320 F to at least +750 F.

Performance Characteristics

In recent years significant improvements were made in calibration techniques for shock and vibration transducers. This led to the development of transducers with improved characteristics and permitted experimental

verification of the transducer's performance. Not only do these techniques demonstrate the measurement accuracies attainable; but they also permit the transducer to be calibrated at the same frequencies and accelerations of intended use.

Sensitivity and Frequency Response. The sensitivity [1] of a transducer is the ratio of its output (e.g. voltage) divided by the applied input stimulus (e.g. acceleration, velocity, force, etc.) including the phase shift of the output from the input.

The input used depends upon the type of transducer. The sensitivity of an accelerometer is the output divided by the applied acceleration. It is usually expressed as volts/ g for self-generating accelerometers. Certain accelerometers employ transducing elements which require electrical excitation. Their sensitivity is determined by dividing the output by the applied acceleration and input excitation voltage, i.e. volts/volt/ g . The sensitivity of velocity pickups is output divided by velocity; force pickups, output divided by applied force. Usually, the units are volts/in./sec and volts/lb for velocity and force pickups, respectively.

The reference direction for expressing the phase shift part of the sensitivity is arbitrarily selected. It is common practice to design an accelerometer to produce a positive voltage when the acceleration is directed from the base to the top of the accelerometer. If the positive voltage increases to a maximum at the same instant the maximum positive acceleration is reached, the phase shift is zero degrees. If the voltage maximum were negative for the same applied acceleration, the phase shift would be 180 degrees. More important than the reference direction selected for expressing phase shift is the variation in phase shift with frequency. When the motion is not sinusoidal the output accurately reproduces the applied motion only when the phase shift is zero or varies linearly with frequency. Seismic transducers with near zero damping have zero phase shift throughout their frequency range. If the damping is near 0.65 of critical damping, the phase shift is proportional to frequency. For other values of damping [1], the phase shift is a non-linear function of frequency.

Figure 1 illustrates a typical sensitivity and frequency response calibration on a piezoelectric accelerometer. The calibration was performed from 5 cps to 10,000 cps by the comparison method [2]. The sensitivity of the accelerometer is nearly constant throughout the frequency range with a four percent sensitivity increase near 10,000 cps. The phase shift is zero degrees indicating the accelerometer has near zero internal damping. This experimentally determined frequency response corresponds to the theoretical response for an ideal linear accelerometer [1] used at frequencies up to one-fifth of its resonance frequency. The actual resonance frequency determined from the test results in Fig. 2 is 52,300 cps. Another approved method [1] for

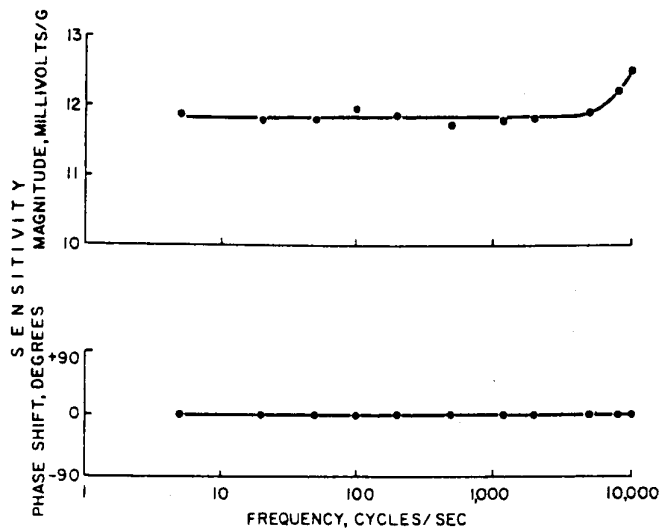


FIGURE 1. CALIBRATION OF A PIEZOELECTRIC ACCELEROMETER.

experimentally determining the resonance frequency is to mechanically excite the accelerometer with a half-sine pulse and to compare the output to the response of an ideal accelerometer [3]. This test, Fig. 3, indicates the resonance frequency is 50,000 cps. Actually, this frequency, the reciprocal of the periodicity of the free oscillations, is the damped natural frequency of the accelerometer. However, since the damping is near zero, the resonance frequency and damped natural frequency occur at almost exactly the same frequency. Close examination of Fig. 3 indicates there is a second resonance near 105,000 cps present. When more than one resonance is present, it is better to rely on the use of an electrodynamic exciter for resonance frequency measurements.

Figure 4 illustrates the sensitivity and frequency

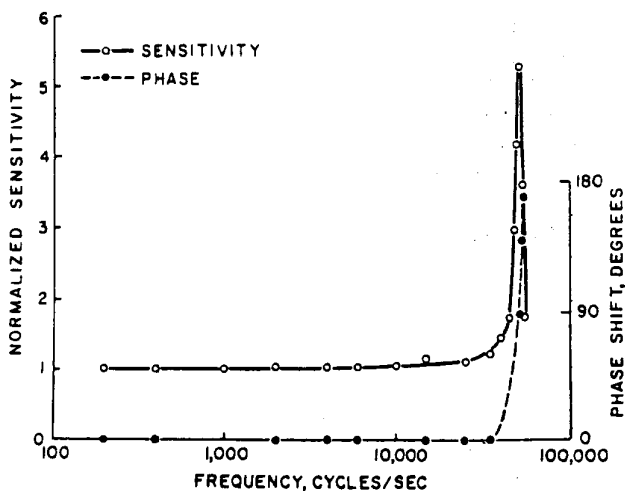


FIGURE 2. EXPERIMENTAL VERIFICATION OF RESONANCE FREQUENCY OF A PIEZOELECTRIC ACCELEROMETER.

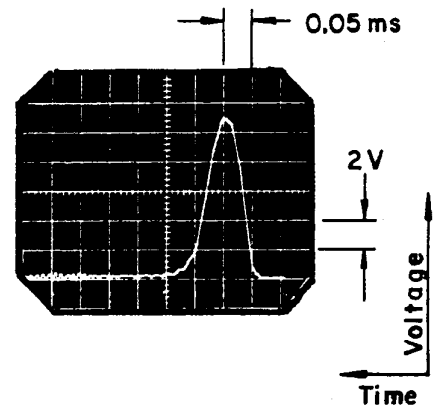


FIGURE 3. RESONANCE FREQUENCY MEASUREMENT BY SHOCK MOTION EXCITATION.

response of a piezoelectric impedance head [4]. This calibration is performed by the comparison method. Both the accelerometer part and force pickup part of the head are calibrated by comparing their outputs to a previously calibrated standard accelerometer. The calibration is performed with a rigid fixture between the standard accelerometer and impedance head. The standard accelerometer is mounted at the same point of attachment of intended use for measuring the impedance of test structures. The applied force is computed from the product of the measured acceleration and the measured dead-weight driven by the force pickup elements. The calibration, Fig. 4, indicates the sensitivities of both the accelerometers and force pickups in the head are constant in the frequency range from 5 cps to 5,000 cps. The phase shifts of both the accelerometers and force pickups are zero throughout the frequency range.

A sensitivity and frequency response calibration performed on a variable resistance accelerometer is illustrated in Fig. 5. The sensitivity is nearly constant at fre-

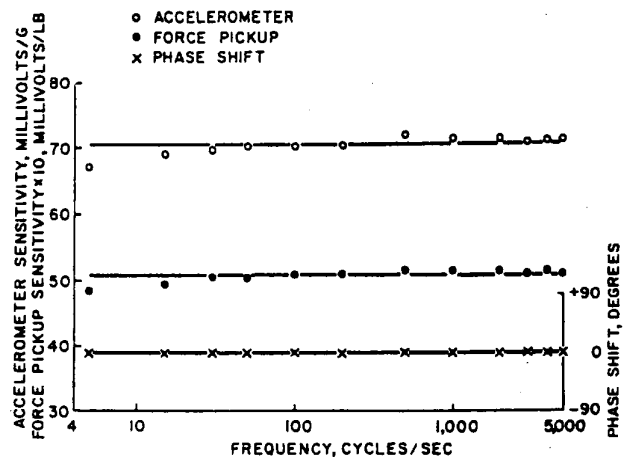


FIGURE 4. CALIBRATION OF A PIEZOELECTRIC IMPEDANCE HEAD.

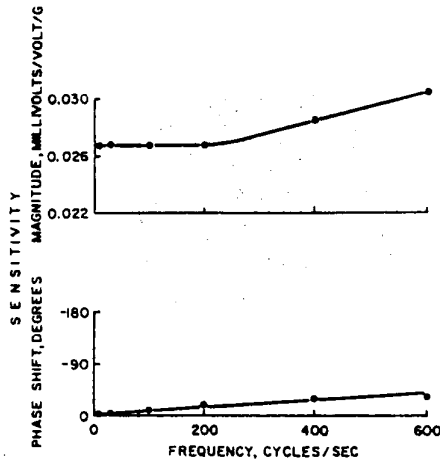


FIGURE 5. COMPARISON CALIBRATION OF A VARIABLE-RESISTANCE ACCELEROMETER [5].

quencies up to 400 cps. The phase shift varies linearly with frequency in the same frequency range. The response corresponds to the theoretical response for the ideal accelerometer having approximately 0.65 of critical damping. Although not indicated in Fig. 5, this type accelerometer responds to zero frequency excitation and can be used for measuring constant accelerations. Zero frequency calibrations can be performed on a tilting support or centrifuge [1]. Other accelerometers capable of zero frequency measurements include differential transformer and variable reluctance types.

All the accelerometers described above are designed to be used below their resonance frequency. Most velocity type pickups are used above their resonance frequency.

A sensitivity and frequency response calibration performed on an electrodynamic velocity pickup with eddy-current damping is illustrated in Fig. 6. The sensitivity of this pickup is constant over a relatively small frequency range. Below 50 cps, the phase shift is a non-linear function of frequency. A pickup with these response characteristics would have distortion in its output when used for shock motion measurements. The distortion is caused by the phase shift non-linearity and lack of low frequency response. An electrodynamic pickup designed with zero damping [5] does not have all these undesirable characteristics. It has zero phase shift and constant sensitivity at frequencies up to 2000 cps.

A sinusoidal motion calibration performed throughout the above acceleration and frequency ranges suffices to demonstrate the suitability of a transducer for most vibration and shock applications. Amplitude linearity calibrations up to 100 g are easily performed on sinusoidally excited resonant beams [4]. Resonant rods are also used at higher accelerations [6] but calibrations at extremely high accelerations within the operating frequency range of the transducer become difficult to per-

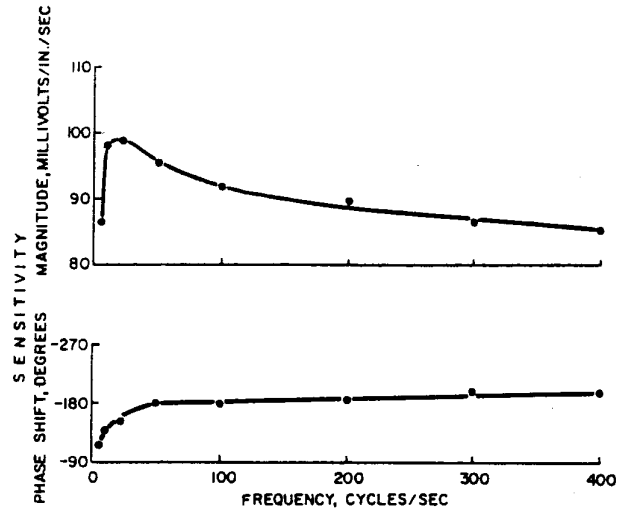


FIGURE 6. CALIBRATION OF AN ELECTRODYNAMIC VELOCITY PICKUP.

form. Also a wide range sinusoidal frequency response calibration will demonstrate that a pickup will measure long duration shock motions without distortion. It is necessary for the calibration to be performed at all frequencies which correspond to the significant Fourier components which represent the shock motion. However, for some shock motions of very short duration and small rise times a calibration to 10,000 cps will not suffice. If the acceleration or frequency ranges covered in the sinusoidal calibration does not correspond to those present in the shock pulse, a shock motion calibration is recommended.

Shock Motion Calibration. Figure 7 illustrates shock motion calibration results obtained on a ballistic type machine [7]. A steel ball is allowed to impact an anvil to which the accelerometer is attached. The padding on the impact surface and the anvil size determine the peak acceleration and pulse duration. The calibration is completed by numerically integrating the accelerometer output and measuring the anvil velocity due to impact. The sensitivities given below each peak acceleration in Fig. 7 indicates this accelerometer is amplitude linear up to at least 10,000 g and is capable of measuring extremely short duration pulses without distortion. The estimated error of this calibration method does not exceed ± 5 percent.

Temperature Response. The temperature response of a piezoelectric accelerometer is illustrated in Fig. 8. The sensitivity is nearly constant from -320 F to room temperature. At high temperatures, the sensitivity decreases to -5 percent near 500 F and near -20 percent at 750 F. This accelerometer would be suitable for making accurate shock and vibration measurements in many applications where extreme temperatures are encountered. In addition to measuring the sensitivity of piezoelectric accelerometers, their electrical resistance

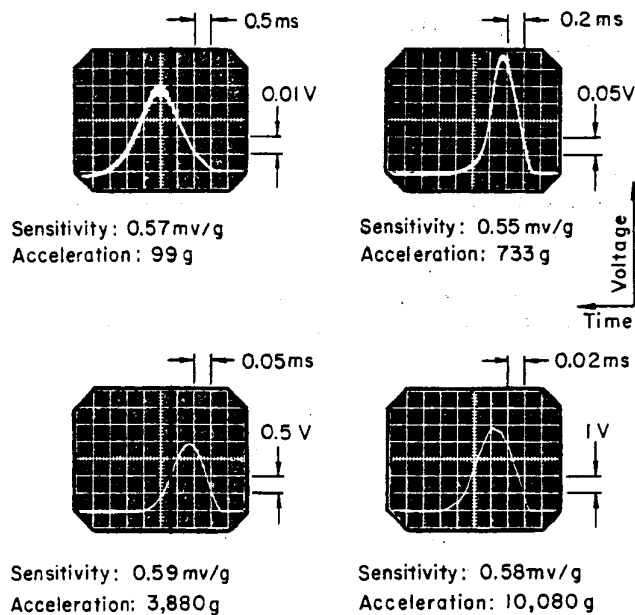


FIGURE 7. SHOCK MOTION CALIBRATION PERFORMED WITHOUT THE USE OF FILTER CIRCUITS.

is measured at high temperatures to insure good response at low frequencies. The resistance of some piezoelectric materials decreases to 100,000,000 ohms at elevated temperatures [8]. Such low resistances affect the frequency response at low frequencies. The effect of low resistance can be minimized by using more capacitance across the accelerometer output terminals.

Damping changes as a function of temperature can adversely affect the frequency response of a transducer at the extremes of the operating frequency range. Since piezoelectric accelerometers have near zero damping their temperature response at high frequencies should be unaffected by changes in damping, if any occurs. This justifies the selection of a low frequency for performing a temperature response calibration. At the present time, laboratory temperature response calibrations are difficult to perform at frequencies much above 500 cps. This calibration limitation is not a serious problem because the operating frequency range of many transducers whose damping is affected by temperature [9] is below 500 cps. Transducers designed to have appreciable damping should have their temperature response calibration performed at frequencies throughout their normal operating frequency range. Frequency response curves obtained on oil and gas damped accelerometers [10, 11] indicate large changes in damping at temperatures between -50 F and 200 F.

Effect of Transducer on Structure. The mechanical impedance of seismic transducers should be considered when testing structures that have relatively low mechanical impedance. The effect of the transducer on the motion of the structure is given by a form of Norton's theorem [12]

$$v_t = v_o \frac{Z_s}{Z_s + Z_t} \quad (1)$$

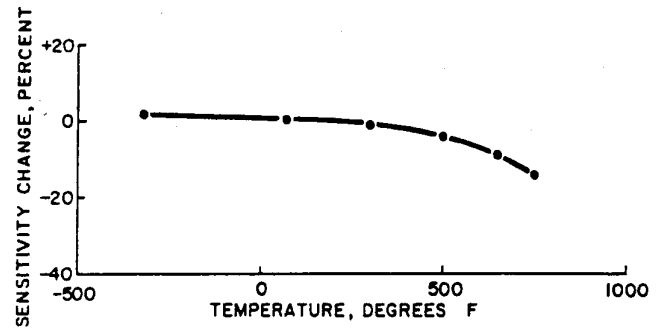


FIGURE 8. TEMPERATURE RESPONSE CALIBRATION OF PIEZOELECTRIC ACCELEROMETER AT 50 CPS.

where v_t = velocity of the structure with transducer attached

v_o = velocity of the structure without transducer attached

Z_s = mechanical impedance of the structure at point transducer is attached

Z_t = mechanical impedance of the transducer.

The motion of the structure is not significantly altered as a result of attaching the transducer if Z_t is small compared to Z_s . Within their operating frequency range, the mechanical impedance, Z_t , of most seismic transducers is $j\omega w/g$ where w is the weight of the transducer (less weight of spring and mass elements for transducers used above their resonance frequency). For most inductive pickups, the weight is at least 0.2 pound and the corresponding mechanical impedance is significant in some measurement applications. Miniature piezoelectric accelerometers weigh as little as 0.006 pound and do not affect structure motions in most test applications. For structures with an extremely thin section at the point motion measurements must be made, it may be necessary to use proximity pickups, e.g. electromagnetic pickups [10]. In this case no attachment is made to the structure and Z_t is zero.

If fixtures are used between the structure and transducer additional errors may occur. In this case Z_t includes the fixture and transducer mechanical impedances. Not only is Z_t significantly increased; but a poor fixture design can result in a resonant condition not present in the structure. This would be the case if the phase angle of Z_t were 180 degrees different from Z_s . As a result the structure motion is greatly changed due to the fixture and transducer. Also, the motion experienced by the transducer is different than that of the structure at the point of fixture attachment.

Mechanical Impedance Measurements

The mechanical impedance at a point on a structure is the complex ratio of the force divided by the velocity.

Both the magnitudes and relative phase angle of the force and velocity are measured. The present trend is to use impedance heads incorporating piezoelectric accelerometers and force pickups to measure mechanical impedance indirectly. Actually, these heads measure the apparent weight, ratio of force in pounds to acceleration expressed in g 's. Since most measurements are made with sinusoidal motion, the mechanical impedance is easily determined from the apparent weight. The mechanical impedance may be computed by multiplying the apparent weight magnitude by ω/g and adding 90 degrees to the apparent weight phase angle. The advantages of using piezoelectric elements include wide operating frequency range, no phase shifts, high outputs, high stiffness and low apparent weight of the head itself. These characteristics permit the measurement of structural impedances over large impedance ranges.

Figure 9 illustrates the range of impedances obtained on an aluminum beam $\frac{3}{4}$ in. \times 3 in. \times 36 in. with a piezoelectric impedance head. The ratio of force and acceleration measurements are plotted using the coordinate lines with the positive slope. The graph paper makes the conversion computation to the magnitudes of the mechanical impedance and dynamic stiffness directly by using the horizontal and negative slope coordinates, respectively. The dynamic stiffness is the ratio of the force to displacement.

The effect of the impedance head on the measured apparent weight is given by a simplification of Norton's theorem [13, 14]

$$W_{33} = W_{12} - W_{22} \quad (2)$$

where W_{33} = point apparent weight of the structure

W_{12} = transfer apparent weight of acceleration at 2 resulting from force at 1 with test structure attached

W_{22} = point apparent weight of impedance head and fixture with test structure removed.

This simplification of Norton's theorem results from the assumption that the point apparent weight and transfer apparent weight of the impedance head and fixture combination with test structure removed are equal. This assumption is justified for the head and fixture used in the calibration given in Fig. 4 since the response is flat throughout the operating frequency range. The impedance head accelerometer in the test with the structure attached, points 2 and 3 connected in Fig. 10, measures the motion in the head rather than that at point 2. It is necessary for these two motions to be identical in order to use (2) accurately. It is expected that this condition can be met in most tests by careful fixture design. The fixture must be rigid at all frequencies used in testing each particular structure. The mechanical impedance test is completed by subtracting the head-structure apparent weight, W_{22} , from the apparent weight, W_{12} ,

measured by the impedance head.

In tests where the apparent weight of the structure [15] is comparable or small in relation to W_{22} , it is important to make this correction. On the other hand, the correction is relatively unimportant for other structures. For example, if this correction were applied in the case of the beam in Fig. 9, only a slight shift in the resonance frequency values would result.

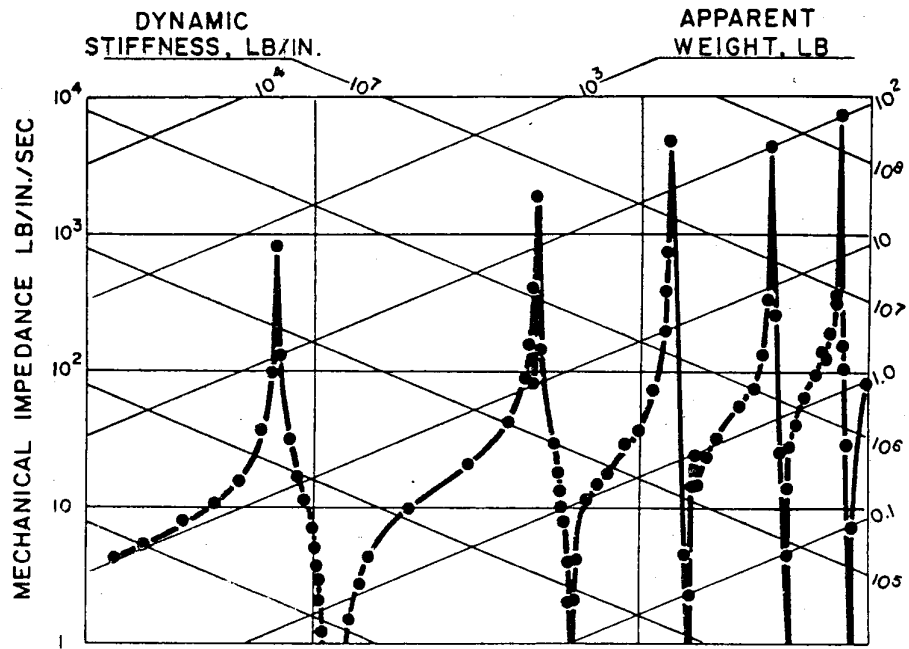
The use of an accelerometer as the motion sensing transducer in the impedance head simplifies the procedure of correcting for W_{22} . Not only is it possible to maintain W_{22} small, but, it is constant throughout the operating frequency range. This means that it should be possible to make the correction automatically by electronically subtracting a portion of the accelerometer signal from the force pickup signal.

The factors limiting the range of impedance measurements are determined by the characteristics of the head and the structure. For small impedances, the accuracy diminishes as the structure impedance becomes small compared to W_{22} . For measuring large impedances, it is easy to design the head so that the stiffness between the force and accelerometer elements is extremely large. The limiting stiffness is determined by the area of contact of the fixture attached to the structure and the characteristics of the structure itself. The annular pressure distribution [16, 17] in the structure may result in a stiffness less than the end stiffness of the head itself. It is also desirable to keep the fixture contact area small in order to minimize local stiffening of the structure.

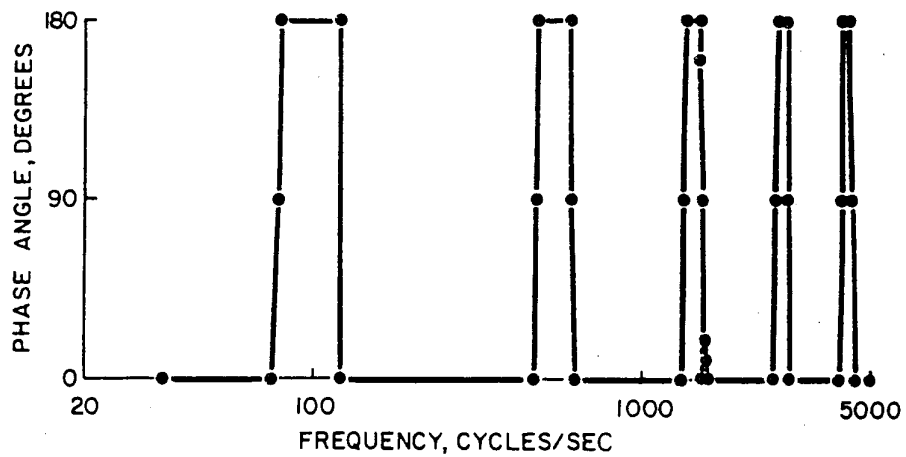
The accuracy near resonances and antiresonances depends upon the structure's impedance and the force available from the exciter providing the driving force. Near resonances, the impedance is small and the portion of the force applied to the structure may be small and therefore, difficult to measure accurately. Near antiresonances, the motion of the structure tends to zero and the signal to noise ratio in the accelerometer output decreases. In addition to the voltage amplifier noise on the accelerometer signal, distortion in the motion of the structure may result from the harmonic distortion in the power amplifier driving the exciter. The power amplifier distortion must be extremely small to prevent motion from occurring at resonance frequencies of the structure which correspond to integer multiples of the driving frequency. It is possible to control these factors in the laboratory, for example the test in Fig. 9. However, in certain applications it may be necessary to use filter circuits on the force and accelerometer outputs.

Vibration Measurements

Periodic vibrations frequently occur in structures and equipment. Sinusoidal testing in the laboratory is used



(A)



(B)

FIGURE 9. MECHANICAL IMPEDANCE OF A BEAM MEASURED WITH AN IMPEDANCE HEAD. (A) THE MAGNITUDE OF THE IMPEDANCE, APPARENT WEIGHT, OR DYNAMIC STIFFNESS IS INDICATED BY THE APPROPRIATE SCALES. (B) THIS PLOT IS THE PHASE ANGLE OF THE APPARENT WEIGHT [4].

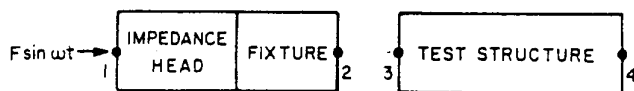


FIGURE 10. DIAGRAM OF TEST SETUP FOR MAKING MECHANICAL IMPEDANCE MEASUREMENTS.

for determining the dynamic characteristics of equipment and evaluating its capability to withstand certain vibration environments. Generally, the above described performance characteristics and calibrations determine which transducer should be selected for a particular field or laboratory vibration measurement. Reasonable care should be taken in rigidly attaching the pickup on a machined surface. In some cases it is desirable to use insulated studs or cement mounting techniques. In addition to mechanical advantages, this is desirable in certain applications to prevent ground loops and minimize electrical noise.

Random vibrations result from the turbulent mixture of gases in missiles and in the exhaust of jet engines. Although the vibrations are aperiodic, the principle requirements of the transducer are the same as in the case of other types of vibration measurements. The acoustic environment associated with this type of vibration is severe and may produce spurious outputs from the transducer. Careful design in the transducer [18] minimizes this effect.

Special auxiliary equipment [19, 20] is required to analyze the spectra of random vibrations. Finally, to simulate random vibrations in the laboratory requires rather elaborate electronic systems [21] to control the acceleration level.

In field testing, one of the important practical problems is in-place gain calibration of entire vibration measuring systems. This is accomplished by inserting a voltage signal in series with the transducer [22]. In addition to adjusting the system gain to obtain desired record height, this calibration serves to discover unintentional short and open circuits, error in the connections of auxiliary equipment, and other system malfunctions.

Shock Measurements

Shock motions impose severe requirements on the instrumentation system. Most shock motions occur in the range from 0-10,000 g with pulse durations between 50 microseconds and 100 milliseconds. In addition to performing calibrations to demonstrate the amplitude linearity, frequency response, phase shift response, and temperature response characteristics, care must be used in selecting pickups and auxiliary instruments in order to accurately measure the shock motion.

The high frequency limitation is generally imposed by the transducer since the response of most amplifiers is flat at frequencies much higher than the trans-

ducer's resonance frequency. For half-sine wave pulses the ratio of the transducer's natural period to pulse duration should be less than one-fifth to prevent resonance excitation [3]. For square-wave pulses, the ratio should be smaller. This requirement can be met with suitable accelerometers. However, mechanical excitation techniques [1] should be used to give experimental evidence of the resonance frequency. The resonance frequency is usually significantly lower than that determined by theoretical [23] considerations or by electrical techniques [24].

The low frequency limitation is determined by the combined characteristics of the pickup and auxiliary electronics. In order to prevent distortion in the indicated pulse and false negative traces, it is necessary to have flat frequency response at very low frequencies. The reciprocal of the frequency where the response is down 3 db should be at least 20 times the pulse duration [25]. This requirement is satisfied by accelerometers having zero frequency response and by selecting suitable amplifiers [26] for use with piezoelectric accelerometers.

In many applications, motions are present at frequencies higher than the significant components of the shock pulse. Usually these motions result from resonances in the test structure or in the test machine [27]. These motions may have little engineering significance and tend to mask the output in the transducer due to the pulse. For this reason it is frequently desirable to use low pass filters. Filters must be carefully selected [28, 29] to eliminate filter ringing and to assure linear phase shift for all significant Fourier components of the shock pulse. Accelerometers and oscillographic galvanometers with near 0.65 of critical damping also filter out high frequency components. In addition to linear phase response, these instruments should be selected to provide sufficiently high frequency response to accurately reproduce the shock pulse.

In some applications extremely high acceleration shock motions are encountered. Generally the pulse durations are shorter for higher accelerations. Piezoelectric accelerometers are frequently employed because of their ruggedness and good high-frequency characteristics. Shock motion calibrations at high accelerations with pulse durations in the microsecond region will demonstrate that the accelerometer is amplitude linear and free of resonance frequency excitation. Even after it is demonstrated that the accelerometer possesses all the required amplitude and frequency characteristics, spurious outputs are possible [30]. It is necessary to use low-noise cable and carefully position it to prevent false signals and breakage of the cable itself. Zero shifts in the output may also occur. These are identified by a definite positive or negative shift in the output at the termination of the shock motion. These shifts generally persist for a much longer period of time than that

associated with undershoot due to insufficient low frequency response. In addition to improper selection and misuse of amplifiers, filters, and cables [31], the possible causes for zero shifts include over-stressing the crystal in the accelerometer. High loads applied for an appreciable period of time may cause inelastic stresses in the crystal which relax over a finite period of time during which the zero-shift output occurs. To eliminate shifts at high accelerations requires careful accelerometer design.

Summary

Shock and vibration instrumentation is suitable for making accurate measurements over wide ranges of frequency, temperature, and acceleration. Recent improvements in calibration techniques now make it possible to experimentally verify the performance characteristics of these instruments.

The use of mechanical impedance heads is developing a new field in vibration measurements. The use of these instruments should lead to a better understanding of the dynamic behavior of structures and permit more realistic testing of equipment.

Shock motion testing requires that the entire measurement system possess wide frequency response and linear phase shift characteristics. Instrumentation systems are available for measurements at extremely high accelerations over a wide range of pulse durations and rise times.

Acknowledgments

N. Der performed most of the calibrations and W. Nazarenko prepared the figures. Their work and the contributions of others are appreciated.

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