RELIABILITY OF A SINGLE-DEGREE-OF-FREEDOM MECHANICAL SYSTEM

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SUMMARY

A method is presented to approximate the inherent reliability of a single-degree-of-freedom system. The transfer function is used as a tool to define mechanical reliability as a function of mechanical failure modes. Primary application of the technique is to new transducer designs for initial design reviews.

INTRODUCTION

Missile borne equipment must be highly reliable and rugged. Measuring systems that possess both of these qualities are sometimes difficult to provide and even more difficult to demonstrate. Small components such as transducers tend to have a relatively large number of failure modes and their use is invariably dictated by tight accuracy requirements. This accuracy requirement is going to grow progressively tighter.

Comparatively little work has been done to evaluate transducer reliability. The technique developed here has been applied to one type of transducer. This transducer (basically a single-degree-of-freedom system) is evaluated by analyzing the transfer function of the design. A failure mode analysis is then applied and relative frequency of occurrence of the modes is established from data and engineering judgement. The effect of the various failure modes on the transfer function is then approximated. The reliability can then be determined using a statistical technique.

PROCEDURE

The purpose of this paper will be in developing this concept beyond the present limits of electronic components to the field of electro-mechanical components. Particularly, the field of electro-mechanical transducers which exhibit a single-degree-of-freedom characteristic. Reliability, aside from being a philosophy and design concept, is also a mathematically definable probability. The standard definition of reliability stated briefly is the probability of performing satisfactorily a given task for a given length of time.

In order to account for both mechanical and electrical failure modes, a model must be developed that will include a failure mode of either type. A simple model which will perform this function is:

Over-all Reliability = Electrical Reliability x Mechanical Reliability

From such a model, precise definitions of electrical and mechanical failures are required. This model is satisfied by the following definitions:

1. Electrical failure: any failure that will degenerate the electrical integrity of the system.

2. Mechanical failure: any failure that affects the parameters of the system transfer function.

One basic assumption in this model that is implied in the definitions of electrical and mechanical failures stated above is that the electrical and mechanical failures are independent events. (Independent events in this case means that an electrical failure will not cause a mechanical failure, nor will a mechanical failure cause an electrical failure. Although this is not completely true, it will be assumed for convenience.) When equipment failure rates are plotted from initial production to field failure, it is seen that equipment follows a constant-failure-rate during part of its life, which is the important concept here. This useful life is also known as the normal operating portion, Poisson portion and the random-failure portion as well as the constant rate portion. This failure rate curve of equipment describes the classic bathtub curve of reliability engineering. Figure 1.

The statistical frequency distribution that describes the constant-failure-rate portion of the bathtub curve is the exponential distribution:

$$R = e^{-\lambda t}$$

where

λ = failure rate

This statistical distribution mathematically defines the probability of a unit failing to be constant during any given time interval of constant length. In other words, when the equipment is operating during its useful life (no parts are wearing out), there is the same probability of the equipment failing in the first 10 seconds of operation as there is in the last 10 seconds of operation -- assuming that there is no failure up to that time.

Electrical Reliability

The electrical reliability has been defined in terms of the system electrical integrity. Thus, knowledge of the equivalent circuit of the electro-mechanical system is required. The equivalent circuit of an electro-mechanical transducer can essentially be manipulated to the form where standard statistical techniques may be applied to determine the reliability.

For example, consider a charge generating transducer with the configuration of Figure 5. The parameters that will affect the electrical reliability are:

- R = resistance
- C = capacitance
- G = charge generator
- X = electrical connections

The detailed evaluation of the electrical reliability will not be included since the techniques may be found in such texts as references 1, 2 and 3.

The electrical reliability is:

$$R = e^{-(\lambda_R + \lambda_C + \lambda_G + 10/3\, \lambda_X)t}$$

Mechanical Reliability

A mechanical failure was previously defined as a failure that affects one or more parameters of the transfer function. The equation of motion for a single-degree-of-freedom mechanical system illustrated in Figure 2 is:

$$\ddot{X} + c/m\, \dot{X} + k/m\, X = Y_o\, f(\omega) \tag{1}$$

where

- m = effective mass
- c = damping
- k = spring constant
- Y_o = excitation amplitude
- X = (response function) displacement of mass relative to housing.

$f(\omega)$ = sinusoidal function

The solution of the equation for an excitation function $Y_o\ f(\omega)$ is:

$$X = Y_o\, f(\omega)\, \frac{\omega^2/\omega_n^2}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\, c/c_c\, \omega/\omega_n\right)^2}} \tag{2}$$

where

- c_c = critical damping
- ω_n = natural frequency = $(k/m)^{1/2}$

From the solution of the equation of motion, the system transfer function can be written:

$$\mathcal{L} = \frac{X\, \omega_n^2}{Y_o\, f(\omega)\, \omega^2} = \frac{1}{\sqrt{\left(1-\omega^2/\omega_n^2\right)^2 + \left(2\, c/c_c\, \omega/\omega_n\right)^2}}$$

When the displacement amplitude of the mass relative to the case is proportional to the applied acceleration amplitude, the instrument is an accelerometer.

The transfer function derived above from the solution of the equation of motion must then be evaluated in terms of system parameters, namely, the effective mass, m, the damping, c, and the spring constant, k. It should be noted, however, that failure modes may alter the excitation function as well as the parameters. The transfer function is then re-written in terms of m, c and k.

$$\mathcal{L} = \frac{1}{\sqrt{\left(1-\frac{m\omega^2}{k}\right)^2 + \left(4\, c^2/c_c^2\, \frac{m\,\omega^2}{k}\right)}}$$

This function is the theoretical response characteristic of the single-degree-of-freedom system in terms of the parameters, m, c and k.

The final steps in the mechanical reliability analysis are to determine the mechanical failure modes, the parameters affected by these modes and the frequency of occurrence for each. This may be done either experimentally or by analytical approximation based on previous designs and field failure data. It can be seen by observation of the transfer function that the system has a definite frequency characteristic. This characteristic is shown in Figure 3. It is obvious that the system response is approximately proportional to the effective mass, inversely proportional to the spring constant, k, and the damping, c. Thus, any changes in the parameters will cause direct operational variations in the response.

Example

Consider a simple single-degree-of-freedom transducer that has three mutually exclusive mechanical failure modes. The only operational specification is frequency response in the range of 5 to 4000 cps. The specification limit is ±3% of the theoretical response. The following values for system parameters are assumed:

- $k = 1.1 \times 10^6$ lb/in
- $c/c_c = 0.01$
- m = 0.0125 lb
- f_n = 30,000 cps

Assume also that the three failure modes have been shown analytically to cause the changes given in Table 1. The assumed failure rate of each failure mode is included.

Table 1 - Mechanical Failure Modes

Failure Modes	Attributable Change in Parameter	Failure Rate % / hr.
I	k decreases 60%	0.0002
11	c/c_c increases 100%	0.0001
111	m decreases 10%	0.0001

When the failure modes are applied individually to the transfer function, the variations of response that result are shown in Figure 4.

It can be seen that every failure mode does not cause a change in the transfer function greater than the specification limits of ±3%. In particular, only the first failure mode (k decreases 60%) actually causes the transfer function to exceed the specification limits. Even though there is more than one mode of failure present in the design, it has been shown that only one is severe enough to cause a mechanical failure. Assume the failure rate for this failure mode to be constant; then the mechanical reliability of the transducer is:

$$R_m = e^{-\lambda_1 t}$$

where

$$\lambda_1 = \text{failure rate of mode 1} = 0.0002$$

Hence, it has been shown that given failure modes and their associated frequency of occurrence, the mechanical reliability can be determined from the transfer function response to the failure modes.

CONCLUSIONS

It has been shown that the reliability of an electro-mechanical transducer is the product of the electrical reliability and the mechanical reliability. Particularly, that the mechanical reliability can be estimated by evaluating the effect of mechanical failure modes on the transfer function of the product. The results indicate that the method applied to the single-degree-of-freedom mechanical system may have extensive applications in evaluating the reliability of accelerometers in space and other severe environments. The technique is restricted in accuracy to the accuracy of the transfer function approximation and the mechanical failure mode probabilities. Using computer techniques and statistical testing, the technique should improve in accuracy with additional data. Several assumptions should also be verified in applying the technique:

1. The validity (or non-validity) of the constancy of mechanical failure rates.

2. The limitations of the validity of the transfer function.

The technique has provided a simple tool to apply to new transducer designs for initial design reviews. It can yield good approximation of inherent reliability with a few simple arithmetic operations when given the transfer function. It is believed that extensive testing should be given to the application of the transform technique to complex mechanical systems as well as the simple one considered here.

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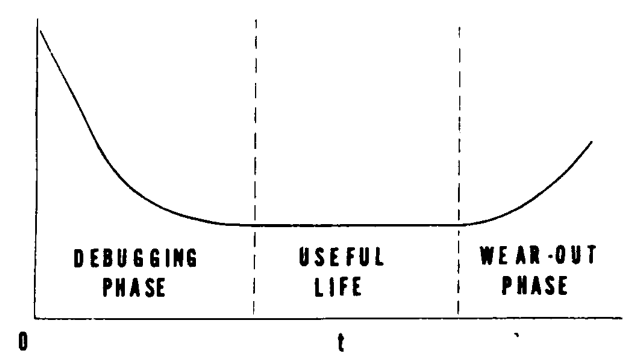
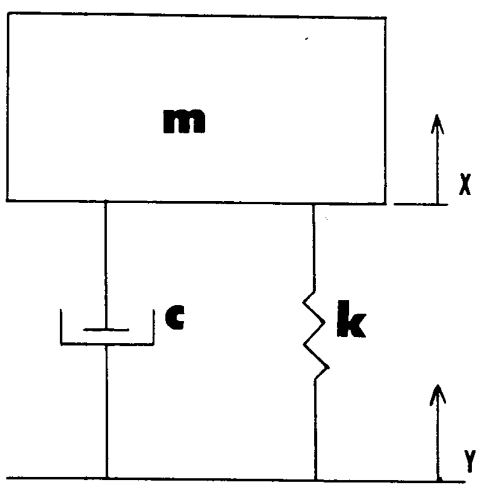


CLASSIC FAILURE-RATE CURVE

figure 1



SINGLE-DEGREE-OF-FREEDOM SYSTEM

figure 2

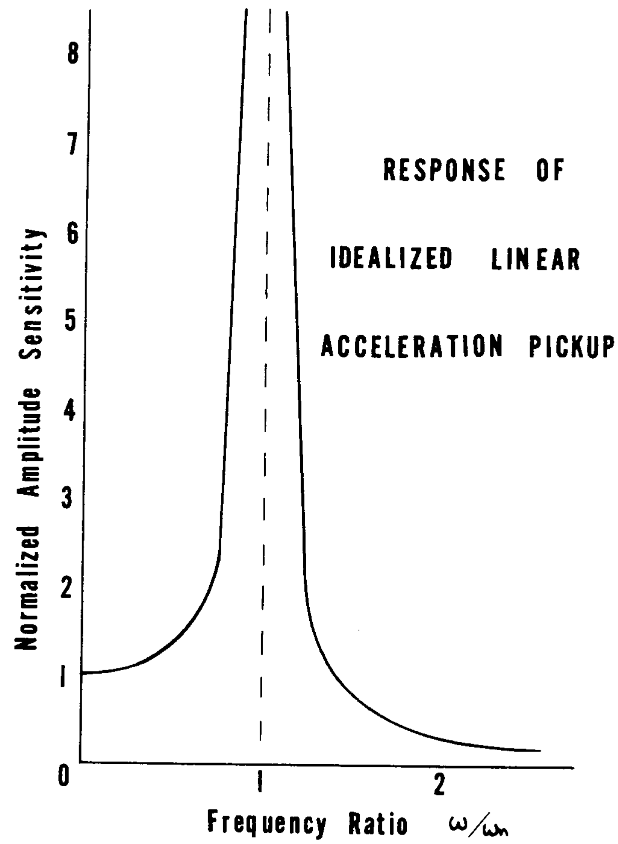


figure 3

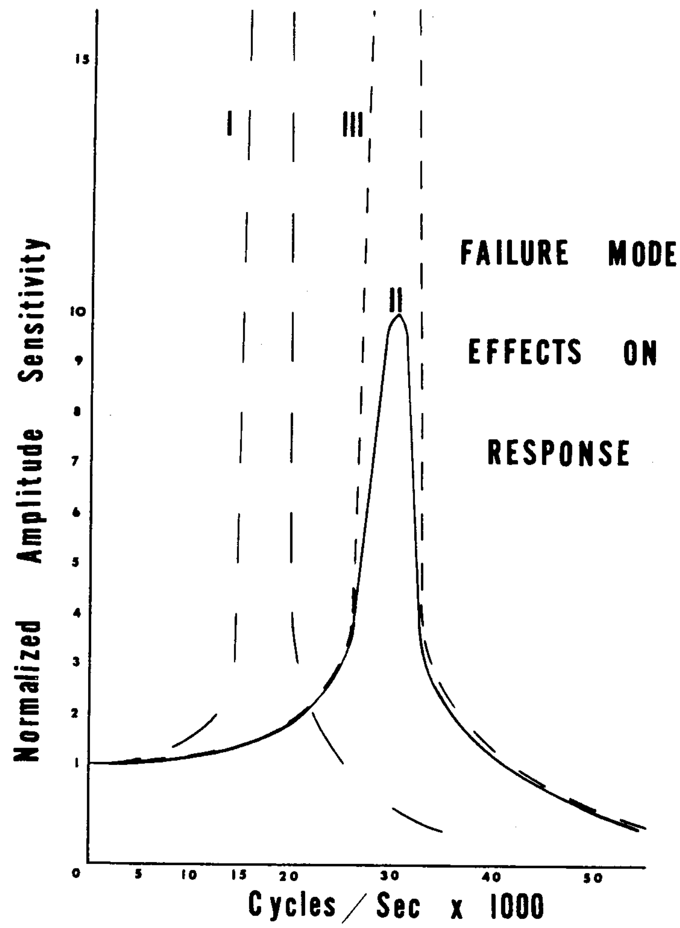
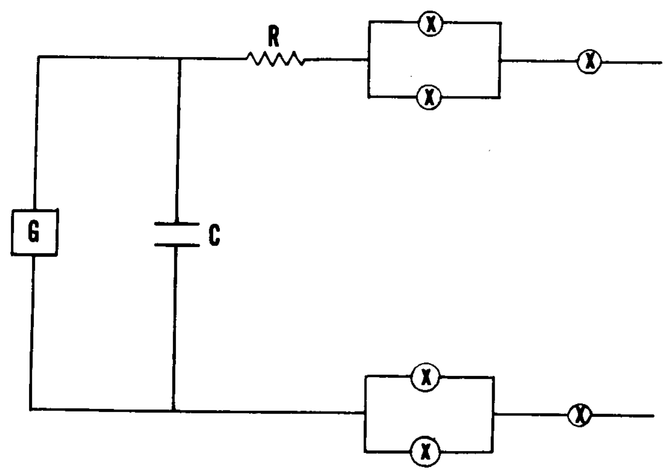


figure 4



EQUIVALENT CIRCUIT

figure 5