TECHNICAL PAPER 214

THE ACCURATE MEASUREMENT OF SHOCK PHENOMENA

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The need to measure shock phenomena has become very important in recent years. In the past much shock information has been in the form of whether a piece of equipment will withstand specified shocks as produced by shock machines such as the "sand drop" or the Jan-S-44 machine. These "machines" are not necessarily repeatable and accurate; therefore, the information obtained using them was whether or not a malfunction was observed in the equipment being tested when it was subjected to the specified shock. This sort of shock testing was adequate when the equipment could be built with an adequate safety factor to make up for the inaccuracies of the testing system.

In the present flight and missile field, size and weight have become very important factors so that large safety factors cannot be built into the systems. Therefore, accurate information is needed as to the nature of shock phenomena that are expected during the operation of the system.

The shocks which are encountered and are of interest can vary from less than one "g" to many thousands of "g" with durations of from a few microseconds to 100 milliseconds or more. This very wide dynamic range makes it very difficult to have a single measuring system that will cover the entire range. The transducer best suited to this requirement is the piezoelectric accelerometer because of its wide dynamic range and its high natural frequency.

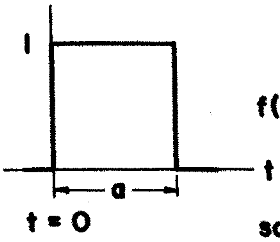
In order to accurately measure the shock phenomenon, a transducer, amplification and signal conditioning system, and readout device must be used.

Each of these will modify the shock pulse somewhat, so that the information obtained at the readout device will have a certain amount of error introduced. In order to determine this error and keep it at a minimum, it is necessary to know the transfer functions of the various sections of the measuring system as well as the time versus amplitude history of the shock pulse being applied.

Reprinted by Endevco Corporation
Presented at the Institute of Environmental Sciences
Sheraton Hotel, Chicago, Illinois, April 11, 12 and 13, 1962

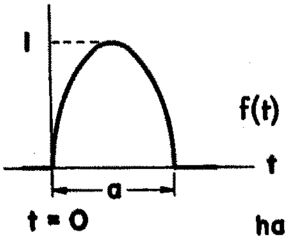
Shock pulses can usually be approximated by one of the following three types: square pulse, half sine pulse, or sawtooth pulse. An equation can be written for each of these pulses that give their time-amplitude history.

The wave shape and equation of each of the fundamental types of shock pulses are shown in Figure 1.



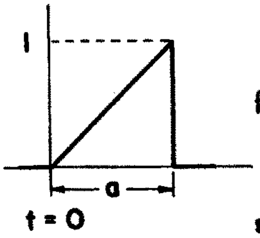
$$f(t) = u(t) - u(t-a)$$

square pulse



$$f(t) = u(t)\,\sin\frac{\pi}{a}\,t - u(t-a)\,\sin\frac{\pi t}{a}$$

half sine pulse



$$f(t) = u(t)\,\frac{t}{a} - u(t-a)\,\frac{t}{a}$$

saw tooth pulse

To determine how these equations are modified by the measuring system, the transfer function of each section of the system must be known. If it is assumed that the error introduced by each section of the system is small, then the total error in the system will be the sum of the errors caused by each section.

An example of the calculation necessary to determine the error introduced in each section is given below.

Assume the shock pulse is a square wave with a pulse duration of "a" seconds. If the electrical analog of this pulse is passed through a section of the system which has a first order low frequency rolloff with a time constant of "T" seconds, then the calculation is as follows:

First make a Laplace Transformation of the equation of the shock pulse:

$$F(s) = \frac{1}{s}\left(1 - e^{-as}\right)$$

Next multiply the function by the Laplace Transform of the transfer function of the section of the measuring system:

$$F(s)' = \frac{1}{s}\left(1 - e^{-as}\right) \times \frac{s}{s + \frac{1}{T}}$$

where: $F(s)'$ is the Laplace Transform of the modified shock pulse.

Take the inverse transform and the equation in the time domain of the modified shock pulse is obtained:

$$f(t)' = u(t)\, e^{-t/T} - u(t - a)\, e^{-(t - a)/T}$$

Figure 2 shows how this modified pulse compares to the original pulse.

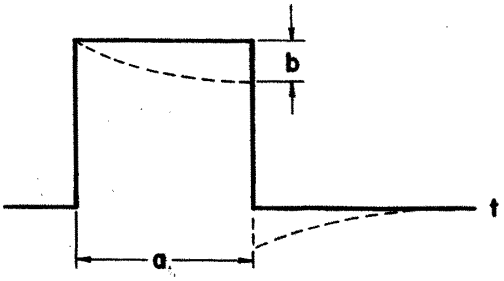


FIGURE 2

The solid line is the original and the dotted line is the modified. As can be seen, the pulse does not hold flat during the entire pulse but decays at an exponential rate. The ratio of the distance "b" to the total height of the pulse is considered the error. From the equation it can be seen that the amount of error is a function of the ratio of $\frac{T}{a}$. The larger this ratio is, the less the error will be. For example, if the ratio is 20, there will be approximately 5% error; if the ratio is 50, there will be approximately 2% error.

The same method of calculation can be used on the other pulse shapes to determine the errors introduced in them. Figure 3 shows a tabulation of the various wave shapes with the errors introduced due to different ratios of $\frac{T}{a}$.

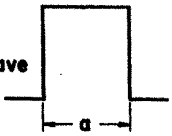
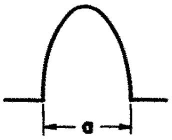
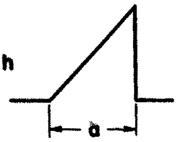
		$\frac{T}{a}$ for approx. 2% error	$\frac{T}{a}$ for approx. 5% error
square wave		50	20
half sine		16	7
saw tooth		16	7

FIGURE 3

The square shock pulse requires the widest dynamic range from the measuring system. This can be seen in Figure 3 by noting the $\frac{T}{a}$ ratios required for the various pulses. Therefore, if the error calculations are made assuming a square shock pulse, then the error introduced in any shock pulse will never be greater than the calculated value.

The calculations shown were for a first order low frequency transfer function. Similar calculations can be made for higher order transfer functions and for high frequency transfer functions.

The Laplace Transforms of the commonly found first and second order transfer functions are given below:

First Order Low Frequency:

$$G(s) = \frac{s}{s + \frac{1}{T}}$$

where: T = low frequency time constant

Second Order Low Frequency:

$$G(s) = \frac{s^2}{s^2 + 2hw_o s + w_o^2}$$

where: w_o = low frequency cutoff frequency

h = low frequency damping factor

First Order High Frequency:

$$G(s) = \frac{1}{s + \frac{1}{K}}$$

where: K = high frequency time constant

Second Order High Frequency:

$$G(s) = \frac{\alpha_o^2}{s^2 + 2k\alpha_o s + \alpha_o^2}$$

where: α_o = high frequency cutoff frequency

k = high frequency damping factor

The second order low frequency transfer function will distort the square pulse in about the same manner as the first order function as long as the cutoff frequency is considerably lower than the lowest frequency component in the pulse. A low frequency damped oscillation will occur after the end of the pulse if the damping factor is less than one. The duration and amplitude of this oscillation will depend on the value of the damping factor.

The high frequency transfer functions have the effect of slowing down the rise time of the shock pulse. If the transfer function is second order, a high frequency ringing at approximately the high frequency cutoff frequency will be superimposed upon the shock pulse. The amplitude and duration of this ringing is dependent on the value of the damping factor. Figure 4a shows the effect of a first order transfer function on a square pulse and Figure 4b shows the effect of a second order transfer function on a square wave pulse.

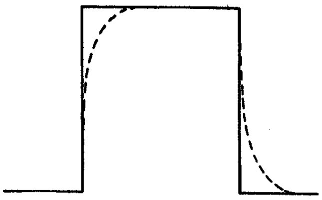


FIGURE 4a

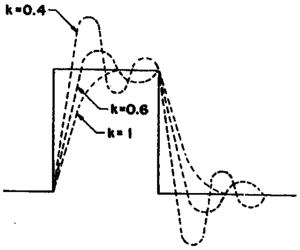


FIGURE 4b

The mathematics involved to make the calculations for higher order functions is extensive so that first order approximations should be made wherever possible.

A useful technique for empirically determining total measuring system error is the use of a simulated signal applied at the input. A square pulse is a good wave shape to use because it is easy to generate and also will show the greatest distortion of the measuring system output. The best way to apply this pulse is to insert a small resistor in series with the ground return of the accelerometer. When this is done, the accelerometer must be electrically isolated by the use of an insulated stud. The signal is then applied across the resistor. Using this method of signal insertion, the whole system can be calibrated including the accelerometer. The only difference which might occur between the square pulse insertion and an actual square shock pulse would be the high frequency ringing which is superimposed on the pulse due to the excitation of the accelerometer resonant frequency.

The piezoelectric accelerometer has both a low and a high frequency transfer function which must be considered. The low frequency is a first order function which is determined by the total source capacity, which includes not only the transducer capacity, but all shunting cable and amplifier input capacity, and the input resistance of the voltage amplifier to which the accelerometer is attached. The high frequency function is determined by the resonant frequency of the transducer and is approximately a second order function with very little damping.

If the low frequency transfer function is not sufficient to meet the requirements of a given system, it can be changed by either choosing an amplifier with a higher input resistance or by adding capacity in parallel with the accelerometer. The adding of parallel capacity will increase the time constant but will also reduce the sensitivity of the system.

The accelerometer is virtually undamped. This provides good high frequency response and minimum phase shift in the region of interest. However, rapid rise times in the shock pulse may excite the accelerometer resonance which will cause a ringing to be superimposed on the basic pulse. With the half sine pulse, if the natural period of the accelerometer is made one fifth of the pulse duration, then the indicated peak will not be more than 10% higher than the actual peak. If less than 10% over shoot is necessary, an accelerometer with a higher resonant frequency must be used.

The amplifier is the next section of the system to be considered. It would be desirable to have a flat frequency response with zero phase shift from zero to over a hundred kilocycles in order that no error is introduced. The high frequency response can be achieved by proper amplifier design. In practice, if the high frequency transfer function of the amplifier is made at least twice the resonant frequency of the accelerometer, no appreciable error will be introduced by the amplifier. However, it is not practical or normally desirable to extend the response to D.C. for amplifiers which require the very high input resistance (1000 megohms) necessary for use with piezoelectric transducers because of the problems of size and zero drift. There is a compromise which must be reached between size requirements and low frequency response.

In general, amplifiers of the type discussed are feedback type. Because of circuit requirements, feedback amplifiers usually have transfer functions which are of second order. Therefore, if calculations are to be made, as described earlier, to determine the error introduced, considerable mathematical effort will be required. To reduce the work, an approximate approach can be used by assuming the transfer function to be of first order with a time constant determined by the 3 db rolloff point. This approximation is quite good since feedback amplifier design requires that the damping factor of the second order function be quite high in order to insure stability under all operating conditions.

Another approach to the problem of minimizing the error is to use a so-called charge amplifier. The piezoelectric accelerometer is basically a charge generating device, that is to say, that at any instant in time, there is a charge across the crystal which is proportional to the acceleration. If this charge can be monitored directly instead of the voltage, then the low frequency transfer characteristics can be eliminated. This is a big advantage because the accelerometer low frequency characteristics are usually the limiting factor in accurately reproducing long shock pulses. The charge amplifier does this and therefore eliminates the transducer low frequency transfer characteristics. Another advantage which the charge amplifier has is that cable capacity does not attenuate the charge signal. This allows the use of long cables without loss of sensitivity.

The charge amplifier consists of a charge converter, which gives a voltage at its output which is proportional to the charge at the input, and a voltage amplifier to raise the voltage level for accurate readout. The voltage amplifier can be made a D.C. amplifier and therefore, the only low frequency transfer function in the whole system up to the readout, is in the charge converter. With the use of electrometer tubes, this transfer function can be made to have a time constant of 30 seconds or even longer if desired.

In many cases only certain frequencies within the shock spectrum are of interest. In this case filtering is introduced in the system to allow only the desired frequencies to pass. It is the purpose of a filter to pass certain frequencies and reject others. Even though the filter passes the desired frequencies without any attenuation, the phase relation of one frequency with respect to another may be changed thus causing an error in the composite wave shape. To eliminate this, the filter should be designed to have constant time delay within its pass band. In this way the phase relation between the frequency components is held and only a time shift of the composite wave shape will occur.

The final section of the system is the readout device. There are various types of devices depending on the type of readout desired. The most common types are: (1) Oscilloscope; (2) Galvanometer; and (3) Magnetic Tape. Another type of readout which is sometimes used is a single peak reading meter. This is used where only the peak value of the shock pulse is of interest. The oscilloscope is probably the most versatile and easy to use. A good quality scope has response from D.C. to over a megacycle so that it will not introduce any errors. It is possible to calibrate the scope directly in "g's" and seconds so that the duration and amplitude of the shock pulse is easily determined. Cameras are also available to photograph the shock pulse for permanent record.

The galvanometer is also a useful readout device. It will make a permanent record on photographic paper of the shock phenomenon. The use of a recording oscillograph with galvanometers has one big advantage; it allows the simultaneous recording of many different points. In this way, it is possible to determine the effects of a shock at various points on a missile or other device. The galvanometer has one other feature; it is a low pass filter. This can be an advantage or a disadvantage depending on what is required at the readout. Unfortunately, the present art of galvanometer design limits the high pass frequency to about 5000 cps. This somewhat limits their use for short duration shocks. The phase characteristic of galvanometers is such that it approximates a constant time delay so that very little phase error is introduced. A disadvantage of the galvanometer is that power is required to drive it and therefore additional power amplification is needed in the amplifier section. This additional power amplification must have frequency response to D.C. to make it compatable with the rest of the system.

Magnetic tape recording is most useful where the shock information is to be fed into a computer for data reduction. In general, the shock information is processed before recording to put it in the form which the magnetic tape and computer can accept. When this is done, the high frequency response is limited in some cases.

If the shock measuring system is to be airborne such as in a missile instrumentation, there is one link in the system which has not been discussed, that is the telemetering link. Normally this would include a voltage controlled oscillator and a telemetering transmitter. The center frequency of the V.C.O. must be chosen sufficiently high so that distortion of the shock pulse does not occur due to the high frequency rolloff of the telemetering system.

SUMMARY

To summarize, each component of the measuring system will in some degree distort the shock pulse being measured. The transducer, in general, is the limiting part of the system. The piezoelectric accelerometer is many times better than other types of transducers which are presently available because of its wide dynamic range and high resonant frequency, and is often better than electronic systems chosen without due care.

A voltage amplifier should be chosen to have high input resistance and sufficiently wide frequency response so that the error introduced is held to a minimum. A charge amplifier is ideal for shock measurement because it eliminates the low frequency transfer function caused by the transducer capacity and amplifier input resistance. If a filter is used, it should be of the constant time delay type so that phase distortion is not introduced.

Finally the readout device should be chosen to give the desired type of readout.

To determine the total error which will be introduced by the system a calculation can be made to determine the error caused by each section of the system and then adding all of the errors together to get the total.

Due to the complexity of working with second order transfer functions, wherever possible an approximation should be made using a first order function.

Figure 5 shows a block diagram of a shock measuring system which incorporates all of the sections discussed.

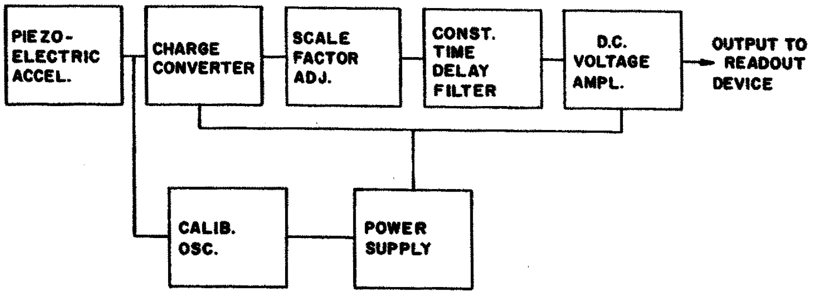


FIGURE 5

It is felt that this type of shock measuring system will give the industry a device which will be very useful in the accurate measurement of shock phenomena.

January 8, 1962

J.C.R.

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